

$$\begin{aligned}
\int e^x \cos x dx &= \int e^x (\sin x)' dx = e^x \sin x - \int (e^x)' \sin x dx = e^x \sin x - \int e^x \sin x dx = \\
&= e^x \sin x - \int e^x (-\cos x)' dx = e^x \sin x + \int e^x (\cos x)' dx = \\
&= e^x \sin x + e^x \cos x - \int (e^x)' \cos x dx = e^x \sin x + e^x \cos x - \int e^x \cos x dx
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int e^x \cos x dx &= e^x \sin x + e^x \cos x - \int e^x \cos x dx \Rightarrow \\
2 \int e^x \cos x dx &= e^x \sin x + e^x \cos x \Rightarrow \int e^x \cos x dx = \frac{1}{2}(e^x \sin x + e^x \cos x)
\end{aligned}$$