CHAPTER 5 Welfare economics and the environment

Welfare economics is the branch of economic theory which has investigated the nature of the policy recommendations that the economist is entitled to make. **Baumol (1977), p. 496**

Learning objectives

In this chapter you will

- learn about the concepts of efficiency and optimality in allocation
- derive the conditions that are necessary for the realisation of an efficient allocation
- find out about the circumstances in which a system of markets will allocate efficiently
- learn about market failure and the basis for government intervention to correct it
- find out what a public good is, and how to determine how much of it the government should supply
- learn about pollution as an external effect, and the means for dealing with pollution problems of different kinds
- encounter the second-best problem

Introduction

When economists consider policy questions relating to the environment they draw upon the basic results of welfare economics. The purpose of this chapter is to consider those results from welfare economics that are most relevant to environmental policy problems. Efficiency and optimality are the two basic concepts of welfare economics, and this chapter explains these concepts as they relate to problems of allocation. There are two classes of allocation problem: static and intertemporal. Efficiency and optimality are central to both. In this chapter we confine attention to the static problem – the allocation of inputs across firms and of outputs across individuals at a point in time. The intertemporal problem – allocation over time – is dealt with in Chapter 11. If you have previously studied a course in welfare economics, you should be able to read through the material of this chapter rather quickly. If not, the chapter will fill that gap.

There are three parts to this chapter. The first states and explains the conditions required for an allocation to be (a) efficient and (b) optimal. These conditions are derived without regard to any particular institutional setting. In the second part of the chapter, we consider how an efficient allocation would be brought about in a market economy characterised by particular institutions. The third part of the chapter looks at the matter of 'market failure' – situations where the institutional conditions required for the operation of pure market forces to achieve efficiency in allocation are not met – in relation to the environment.

PART 1 EFFICIENCY AND OPTIMALITY

In this part, and the next, of this chapter we will, following the usage in the welfare economics literature, use 'resources' to refer generally to inputs to production rather than specifically to extractions from the natural environment for use in production. In fact, in these parts of the chapter, when we talk about resources, or 'productive resources' we will have in mind, as we will often make explicit, inputs of capital and labour to production.

At any point in time, an economy will have access to particular quantities of productive resources. Individuals have preferences about the various goods that it is feasible to produce using the available resources. An 'allocation of resources', or just an 'allocation', describes what goods are produced and in what quantities they are produced, which combinations of resource inputs are used in producing those goods, and how the outputs of those goods are distributed between persons.

In this section, and the next, we make two assumptions that will be relaxed in the third part of this chapter. First, that no externalities exist in either consumption or production; roughly speaking, this means that consumption and production activities do not have unintended and uncompensated effects upon others. Second, that all produced goods and services are private (not public) goods; roughly speaking, this means that all outputs have characteristics that permit of exclusive individual consumption on the part of the owner.

In the interests of simplicity, but with no loss of generality, we strip the problem down to its barest essentials. Our economy consists of two persons (A and B); two goods (X and Y) are produced; and production of each good uses two inputs (K for capital and L for labour) each of which is available in a fixed quantity.

Let U denote an individual's total utility, which depends only on the quantities of the two goods that he or she consumes. Then we can write the utility functions for A and B in the form shown in equations 5.1:

$$\begin{aligned} U^{\mathrm{A}} &= U^{\mathrm{A}}(X^{\mathrm{A}},\ Y^{\mathrm{A}}) \\ U^{\mathrm{B}} &= U^{\mathrm{B}}(X^{\mathrm{B}},\ Y^{\mathrm{B}}) \end{aligned} \tag{5.1}$$

The total utility enjoyed by individual A, denoted U^{A}, depends upon the quantities, X^{A} and Y^{A}, he or she consumes of the two goods. An equivalent statement can be made about B's utility.

Next, we suppose that the quantity produced of good X depends only on the quantities of the two inputs K and L used in producing X, and the quantity produced of good Y depends only on the quantities of the two inputs K and L used in producing Y. Thus, we can write the two production functions in the form shown in 5.2:

$$\begin{aligned} X &= X(K^X,\ L^X) \\ Y &= Y(K^Y,\ L^Y) \end{aligned} \tag{5.2}$$

Each production function specifies how the output level varies as the amounts of the two inputs are varied. In doing that, it assumes technical efficiency in production. The production function describes, that is, how output depends on input combinations, given that inputs are not simply wasted. Consider a particular input combination K_1^X and L_1^X with X_1 given by the production function. Technical efficiency means that in order to produce more of X it is necessary to use more of K^X and/or L^X.

The marginal utility that A derives from the consumption of good X is denoted U_X^{A}; that is, $U_X^{\mathrm{A}} = \partial U^{\mathrm{A}}/\partial X^{\mathrm{A}}$. The marginal product of the input L in the production of good Y is denoted as MP_L^Y; that is, $\mathrm{MP}_L^Y = \partial Y/\partial L^Y$. Equivalent notation applies for the other three marginal products.

The marginal rate of utility substitution for A is the rate at which X can be substituted for Y at the margin, or vice versa, while holding the level of A's utility constant. It varies with the levels of consumption of X and Y and is given by the slope of the indifference curve. We denote A's marginal rate of substitution as $\mathrm{MRUS}^{\mathrm{A}}$, and similarly for B.

The marginal rate of technical substitution as between K and L in the production of X is the rate at which K can be substituted for L at the margin, or vice versa, while holding the level output of X constant. It varies with the input levels for K and L and is given by the slope of the isoquant. We denote the marginal rate of substitution in the production of X as MRTS_X, and similarly for Y.

The marginal rates of transformation for the commodities X and Y are the rates at which the output of one can be transformed into the other by marginally shifting capital or labour from one line of production to the other. Thus, MRT_L is the increase in the output of Y obtained by shifting a small, strictly an infinitesimally small, amount of labour from use in the production of X to use in the production of Y, or vice versa. Similarly, MRT_K is the increase in the output of Y obtained by shifting a small, strictly an infinitesimally small, amount of capital from use in the production of X to use in the production of Y, or vice versa.

With this notation we can now state, and provide intuitive explanations for, the conditions that characterise efficient and optimal allocations. Appendix 5.1 uses the calculus of constrained optimisation (which was reviewed in Appendix 3.1) to derive these conditions formally.

5.1 Economic efficiency

An allocation of resources is said to be efficient if it is not possible to make one or more persons better off without making at least one other person worse off. Conversely, an allocation is inefficient if it is possible to improve someone's position without worsening the position of anyone else. A gain by one or more persons without anyone else suffering is known as a Pareto improvement. When all such gains have been made, the resulting allocation is sometimes referred to as Pareto optimal, or Pareto efficient. A state in which there is no possibility of Pareto improvements is sometimes referred to as being allocatively efficient, rather than just efficient, so as to differentiate the question of efficiency in allocation from the matter of technical efficiency in production.

Efficiency in allocation requires that three efficiency conditions are fulfilled – efficiency in consumption, efficiency in production, and product-mix efficiency.

5.1.1 Efficiency in consumption

Consumption efficiency requires that the marginal rates of utility substitution for the two individuals are equal:

$$\text{MRUS}^A = \text{MRUS}^B \tag{5.3}$$

If this condition were not satisfied, it would be possible to rearrange the allocation as between A and B of whatever is being produced so as to make one better

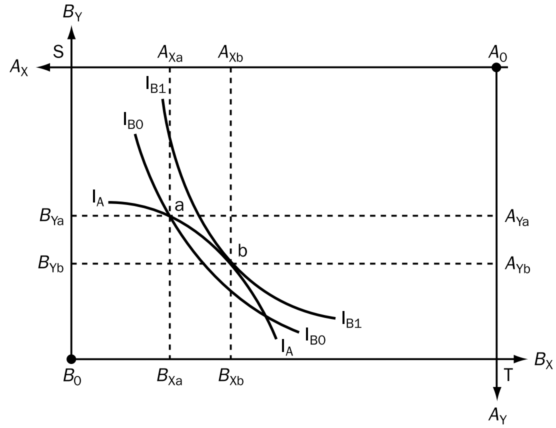

Figure 5.1 Efficiency in consumption

off without making the other worse off. Figure 5.1 shows what is involved by considering possible allocations of fixed amounts of X and Y between A and B.[1] The top right-hand corner, labelled A_0, refers to the situation where A gets nothing of the available X or Y, and B gets all of both commodities. The bottom left-hand corner, B_0, refers to the situation where B gets nothing and A gets everything. Starting from A_0 moving horizontally left measures A's consumption of X, and moving vertically downwards measures A's consumption of Y. As A's consumption of a commodity increases, so B's must decrease. Starting from B_0 moving horizontally right measures B's consumption of X, and moving vertically upwards measures B's consumption of Y. Any allocation of X and Y as between A and B is uniquely identified by a point in the box SA_0TB_0. At the point a, for example, A is consuming A_0A_{Xa} of X and A_0A_{Ya} of Y, and B is consuming B_0B_{Xa} of X and B_0B_{Ya} of Y.

The point a is shown as lying on I_AI_A, which is an indifference curve for individual A. I_AI_A may look odd for an indifference curve, but remember that it is drawn with reference to the origin for A which is the point A_0. Also shown are two indifference curves for B, $I_{B0}I_{B0}$ and $I_{B1}I_{B1}$. Consider a reallocation as between A and B, starting from point a and moving along I_AI_A, such that A is giving up X and gaining Y, while B is gaining X and giving up Y. Initially, this means increasing utility for B, movement onto a

[1] This figure is an 'Edgeworth box'.

higher indifference curve, and constant utility for A. However, beyond point b any further such reallocations will involve decreasing utility for B. Point b identifies a situation where it is not possible to make individual B better off while maintaining A's utility constant – it represents an efficient allocation of the given amounts of X and Y as between A and B. At b, the slopes of $I_A I_A$ and $I_{B1} I_{B1}$ are equal – A and B have equal marginal rates of utility substitution.

5.1.2 Efficiency in production

Turning now to the production side of the economy, recall that we are considering an economy with two inputs, L and K, which can be used (via the production functions of equations 5.2) to produce the goods X and Y. Efficiency in production requires that the marginal rate of technical substitution be the same in the production of both commodities. That is,

$$\text{MRTS}_X = \text{MRTS}_Y \qquad (5.4)$$

If this condition were not satisfied, it would be possible to reallocate inputs to production so as to produce more of one of the commodities without producing less of the other. Figure 5.2 shows why this condition is necessary. It is constructed in a similar manner to Figure 5.1, but points in the box refer to allocations of capital and labour to the production of the two commodities rather than to allocations of the commodities between individuals.[2] At X_0 no capital or labour is devoted to the production of commodity X – all of both resources is used in the production of Y. Moving horizontally to the left from X_0 measures increasing use of labour in the production of X, moving vertically down from X_0 measures increasing use of capital in the production of X. The corresponding variations in the use of inputs in the production of Y – any increase/decrease in use for X production must involve a decrease/increase in use for Y production – are measured in the opposite directions starting from origin Y_0.

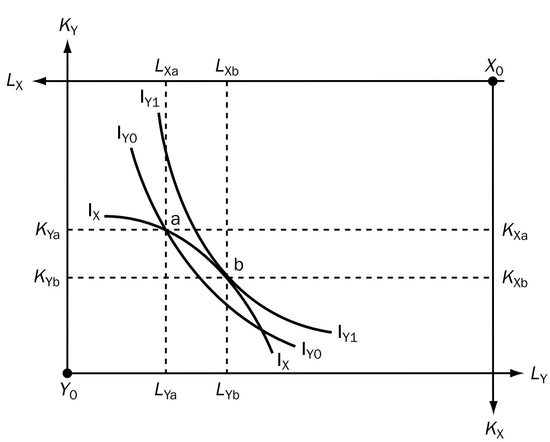

Figure 5.2 Efficiency in production

$I_X I_X$ is an isoquant for the production of commodity X. Consider movements along it to the 'south-east' from point a, so that in the production of X capital is being substituted for labour, holding output constant. Correspondingly, given the full employment of the resources available to the economy, labour is being substituted for capital in the production of Y. $I_{Y0} I_{Y0}$ and $I_{Y1} I_{Y1}$ are isoquants for the production of Y. Moving along $I_X I_X$ from a toward b means moving onto a higher isoquant for Y – more Y is being produced with the production of X constant. Movement along $I_X I_X$ beyond point b will mean moving back to a lower isoquant for Y. The point b identifies the highest level of production of Y that is possible, given that the production of X is held at the level corresponding to $I_X I_X$ and that there are fixed amounts of capital and labour to be allocated as between production of the two commodities. At point b the slopes of the isoquants in each line of production are equal – the marginal rates of technical substitution are equal. If these rates are not equal, then clearly it would be possible to reallocate inputs as between the two lines of production so as to produce more of one commodity without producing any less of the other.

5.1.3 Product-mix efficiency

The final condition necessary for economic efficiency is product-mix efficiency. This requires that

[2] Appendix 5.1 establishes that all firms producing a given commodity are required to operate with the same marginal rate of technical substitution. Here we are assuming that one firm produces all of each commodity.

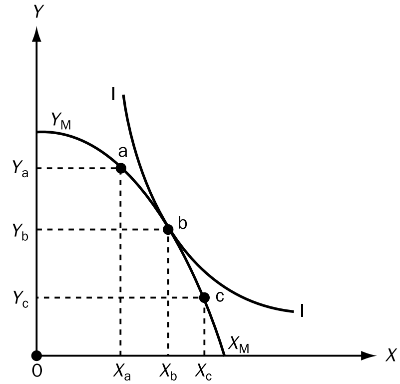

Figure 5.3 Product-mix efficiency

$$\text{MRT}_L = \text{MRT}_K = \text{MRUS}^A = \text{MRUS}^B \tag{5.5}$$

This condition can be understood using Figure 5.3. Given that equation 5.3 holds, so that the two individuals have equal marginal rates of utility substitution and $\text{MRUS}^A = \text{MRUS}^B$, we can proceed as if they had the same utility functions, for which II in Figure 5.1 is an indifference curve with slope MRUS. The individuals do not, of course, actually have the same utility functions. But, given the equality of the MRUS, their indifference curves have the same slope at an allocation that satisfies the consumption efficiency condition, so we can simplify, without any real loss, by assuming the same utility functions and drawing a single indifference curve that refers to all consumers. Given that Equation 5.4 holds, when we think about the rate at which the economy can trade off production of X for Y and vice versa, it does not matter whether the changed composition of consumption is realised by switching labour or capital between the two lines of production. Consequently, in Figure 5.3 we show a single production possibility frontier, $Y_M X_M$, showing the output combinations that the economy could produce using all of its available resources. The slope of $Y_M X_M$ is MRT.

In Figure 5.3 the point a must be on a lower indifference curve than II. Moving along $Y_M X_M$ from point a toward b must mean shifting to a point on a higher indifference curve. The same goes for movement along $Y_M X_M$ from c toward b. On the other hand, moving away from b, in the direction of either a or c, must mean moving to a point on a lower indifference curve. We conclude that a point like b, where the slopes of the indifference curve and the production possibility frontier are equal, corresponds to a product mix – output levels for X and Y – such that the utility of the representative individual is maximised, given the resources available to the economy and the terms on which they can be used to produce commodities. We conclude, that is, that the equality of MRUS and MRT is necessary for efficiency in allocation. At a combination of X and Y where this condition does not hold, some adjustment in the levels of X and Y is possible which would make the representative individual better off.

An economy attains a fully efficient static allocation of resources if the conditions given by equations 5.3, 5.4 and 5.5 are satisfied simultaneously. Moreover, it does not matter that we have been dealing with an economy with just two persons and two goods. The results readily generalise to economies with many inputs, many goods and many individuals. The only difference will be that the three efficiency conditions will have to hold for each possible pairwise comparison that one could make, and so would be far more tedious to write out.

5.2 An efficient allocation of resources is not unique

For an economy with given quantities of available resources, production functions and utility functions, there will be many efficient allocations of resources. The criterion of efficiency in allocation does not, that is, serve to identify a particular allocation.

To see this, suppose first that the quantities of X and Y to be produced are somehow given and fixed. We are then interested in how the given quantities of X and Y are allocated as between A and B, and the criterion of allocative efficiency says that this should be such that A/B cannot be made better off except by making B/A worse off. This was what we considered in Figure 5.1 to derive equation 5.3, which says that an efficient allocation of fixed quantities of X and Y will be such that the slopes of the indifference curves for A and B will be the same. In Figure 5.1 we showed just one indifference curve for A and

Box 5.1 Productive inefficiency in ocean fisheries

The total world marine fish catch increased steadily from the 1950s through to the late 1980s, rising by 32% between the periods 1976–1978 and 1986–1988 (UNEP, 1991). However, the rate of increase was slowing toward the end of this period, and the early 1990s witnessed downturns in global harvests. The harvest size increased again in the mid-1990s, was at a new peak in 1996, and then levelled off again in the late 1990s. It is estimated that the global maximum sustainable harvest is about 10% larger than harvest size in the late 1990s.

The steady increase in total catch until 1989 masked significant changes in the composition of that catch; as larger, higher-valued stocks became depleted, effort was redirected to smaller-sized and lower-valued species. This does sometimes allow depleted stocks to recover, as happened with North Atlantic herring, which recovered in the mid-1980s after being overfished in the late 1970s. However, many fishery scientists believe that these cycles of recovery have been modified, and that species dominance has shifted permanently towards smaller species.

Rising catch levels have put great pressure on some fisheries, particularly those in coastal areas, but also including some pelagic fisheries. Among the species whose catch declined over the period 1976–1988 are Atlantic cod and herring, haddock, South African pilchard and Peruvian anchovy. Falls in catches of these species have been compensated for by much increased harvests of other species, including Japanese pilchard in the north-west Pacific.

Where do inefficiencies enter into this picture? We can answer this question in two ways. First, a strong argument can be made to the effect that the total amount of resources devoted to marine fishing is excessive, probably massively so. We shall defer giving evidence to support this claim until Chapter 17 (on renewable resources), but you will see there that a smaller total fishing fleet would be able to catch at least as many fish as the present fleet does. Furthermore, if fishing effort were temporarily reduced so that stocks were allowed to recover, a greater steady-state harvest would be possible, even with a far smaller world fleet of fishing vessels. There is clearly an inefficiency here.

A second insight into inefficiency in marine fishing can be gained by recognising that two important forms of negative external effect operate in marine fisheries, both largely attributable to the fact that marine fisheries are predominantly open-access resources. One type is a so-called crowding externality, arising from the fact that each boat's harvesting effort increases the fishing costs that others must bear. The second type may be called an 'intertemporal externality': as fisheries are often subject to very weak (or even zero) access restrictions, no individual fisherman has an incentive to conserve stocks for the future, even if all would benefit if the decision were taken jointly.

As the concepts of externalities and open access will be explained and analysed in the third part of this chapter, and applied to fisheries in Chapter 17, we shall not explain these ideas any further now. Suffice it to say that production in market economies will, in general, be inefficient in the presence of external effects.

Sources: WRI (2000), WRI web site www.wri.org, FAO web site www.fao.org

two for B. But, these are just a small subset of the indifference curves for each individual that fill the box SA_0TB_0. In Figure 5.4 we show a larger subset for each individual. Clearly, there will be a whole family of points, like b in Figure 5.1, at which the slopes of the indifference curves for A and B are equal, at which they have equal marginal rates of utility substitution. At any point along CC in Figure 5.4, the consumption efficiency condition is satisfied. In fact, for given available quantities of X and Y there are an indefinitely large number of allocations as between A and B that satisfy $MRUS^A = MRUS^B$.

Now consider the efficiency in production condition, and Figure 5.2. Here we are looking at variations in the amounts of X and Y that are produced. Clearly, in the same way as for Figures 5.1 and 5.4, we could introduce larger subsets of all the possible isoquants for the production of X and Y to show that there are many X and Y combinations that satisfy equation 5.4, combinations representing uses of capital and labour in each line of production such that the slopes of the isoquants are equal, $MRTS_X = MRTS_Y$.

So, there are many combinations of X and Y output levels that are consistent with allocative efficiency,

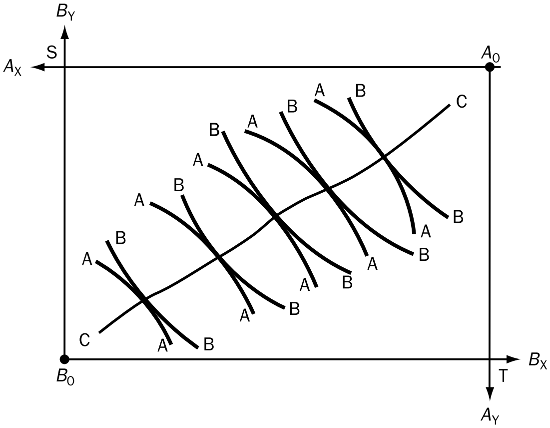

Figure 5.4 The set of allocations for consumption efficiency

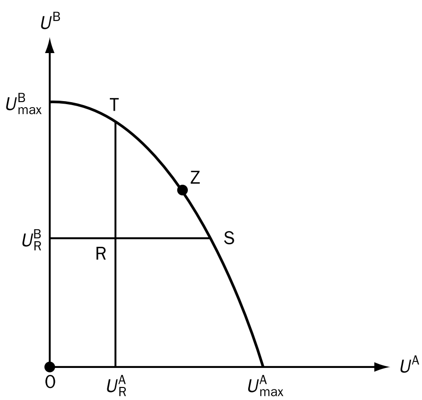

Figure 5.5 The utility possibility frontier

and for any particular combination there are many allocations as between A and B that are consistent with allocative efficiency. These two considerations can be brought together in a single diagram, as in Figure 5.5, where the vertical axis measures A's utility and the horizontal B's. Consider a particular allocation of capital and labour as between *X* and *Y* production which implies particular output levels for *X* and *Y*, and take a particular allocation of these output levels as between A and B – there will correspond a particular level of utility for A and for B, which can be represented as a point in U^A/U^B space, such as R in Figure 5.5. Given fixed amounts of capital and labour, not all points in U^A/U^B space are feasible. Suppose that all available resources were used to produce commodities solely for consumption by A, and that the combination of *X* and *Y* then produced was such as to maximise A's utility. Then, the corresponding point in utility space would be U^A_{max} in Figure 5.5. With all production serving the interests of B, the corresponding point would be U^B_{max}. The area bounded by $U^A_{max}0U^B_{max}$ is the utility possibility set – given its resources, production technologies and preferences, the economy can deliver all combinations of U^A and U^B lying in that area. The line $U^A_{max}U^B_{max}$ is the utility possibility frontier – the economy cannot deliver combinations of U^A and U^B lying outside that line. The shape of the utility possibility frontier depends on the particular forms of the utility and production functions, so the way in which it is represented in Figure 5.5 is merely one possibility. However, for the usual assumptions about utility and production functions, it would be generally bowed outwards in the manner shown in Figure 5.5.

The utility possibility frontier is the locus of all possible combinations of U^A and U^B that correspond to efficiency in allocation. Consider the point R in Figure 5.5, which is inside the utility possibility frontier. At such a point, there are possible reallocations that could mean higher utility for both A and B. By securing allocative efficiency, the economy could, for example, move to a point on the frontier, such as Z. But, given its endowments of capital and labour, and the production and utility functions, it could not continue northeast beyond the frontier. Only U^A/U^B combinations lying along the frontier are feasible. The move from R to Z would be a Pareto improvement. So would be a move from R to T, or to S, or to any point along the frontier between T and S.

The utility possibility frontier shows the U^A/U^B combinations that correspond to efficiency in allocation – situations where there is no scope for a Pareto improvement. There are many such combinations. Is it possible, using the information available, to say which of the points on the frontier is best from the point of view of society? It is not possible, for the simple reason that the criterion of economic efficiency does not provide any basis for making interpersonal comparisons. Put another way, efficiency does not give us a criterion for judging which allocation is best from a social point of view. Choosing a point along the utility possibility frontier is about making moves that must involve making one individual worse off in order to make the other better off. Efficiency criteria do not cover such choices.

5.3 The social welfare function and optimality

In order to consider such choices we need the concept of a social welfare function, SWF, which was introduced in Chapter 3. A SWF can be used to rank alternative allocations. For the two-person economy that we are examining, a SWF will be of the general form:

$$W = W(U^{A}, U^{B}) \tag{5.6}$$

The only assumption that we make here regarding the form of the SWF is that welfare is non-decreasing in U^A and U^B. That is, for any given level of U^A welfare cannot decrease if U^B were to rise and for any given level of U^B welfare cannot decrease if U^A were to rise. In other words, we assume that $W_A = \partial W/\partial U^A$ and $W_B = \partial W/\partial U^B$ are both positive. Given this, the SWF is formally of the same nature as a utility function. Whereas the latter associates numbers for utility with combinations of consumption levels X and Y, a SWF associates numbers for social welfare with combinations of utility levels U^A and U^B. Just as we can depict a utility function in terms of indifference curves, so we can depict a SWF in terms of social welfare indifference curves. Figure 5.6 shows a social welfare indifference curve WW that has the same slope as the utility possibility frontier at b, which point identifies the combination of U^A and U^B that maximises the SWF.

The reasoning which establishes that b corresponds to the maximum of social welfare that is attainable should be familiar by now – points to the left or the right of b on the utility possibility frontier, such as a and c, must be on a lower social welfare indifference curve, and points outside of the utility possibility frontier are not attainable. The fact that the optimum lies on the utility possibility frontier means that all of the necessary conditions for efficiency must hold at the optimum. Conditions 5.3, 5.4 and 5.5 must be satisfied for the maximisation of

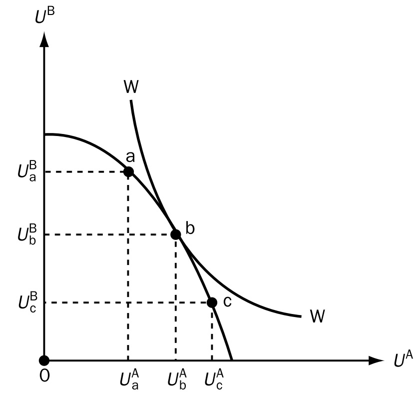

Figure 5.6 Maximised social welfare

welfare. Also, an additional condition, the equality of the slopes of a social indifference curve and the utility possibility frontier, must be satisfied. This condition can be stated, as established in Appendix 5.1, as

$$\frac{W_A}{W_B} = \frac{U^B_X}{U^A_X} = \frac{U^B_Y}{U^A_Y} \tag{5.7}$$

The left-hand side here is the slope of the social welfare indifference curve. The two other terms are alternative expressions for the slope of the utility possibility frontier. At a social welfare maximum, the slopes of the indifference curve and the frontier must be equal, so that it is not possible to increase social welfare by transferring goods, and hence utility, between persons.

While allocative efficiency is a necessary condition for optimality, it is not generally true that moving from an allocation that is not efficient to one that is efficient must represent a welfare improvement. Such a move might result in a lower level of social welfare. This possibility is illustrated in Figure 5.7. At C the allocation is not efficient, at D it is. However, the allocation at C gives a higher level of social welfare than does that at D. Having made this point, it should also be said that whenever there is an inefficient allocation, there is always some other allocation which is both efficient and superior in welfare terms. For example, compare points C and E. The latter is allocatively efficient while C is not, and E is on a higher social welfare indifference curve. The move from C to E is a Pareto improvement

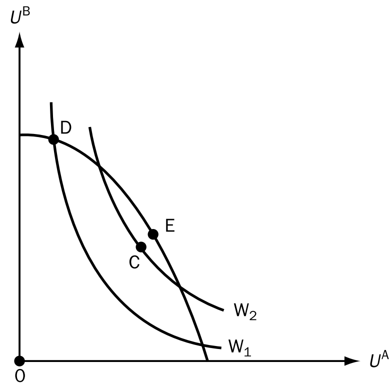

Figure 5.7 Welfare and efficiency

where both A and B gain, and hence involves higher social welfare. On the other hand, going from C to D replaces an inefficient allocation with an efficient one, but the change is not a Pareto improvement – B gains, but A suffers – and involves a reduction in social welfare. Clearly, any change which is a Pareto improvement must increase social welfare as defined here. Given that the SWF is non-decreasing in U^A and U^B, increasing U^A/U^B without reducing U^B/U^A must increase social welfare. For the kind of SWF employed here, a Pareto improvement is an unambiguously good thing (subject to the possible objections to preference-based utilitarianism noted in Chapter 3, of course). It is also clear that allocative efficiency is a good thing (subject to the same qualification) if it involves an allocation of commodities as between individuals that can be regarded as fair. Judgements about fairness, or equity, are embodied in the SWF in the analysis here. If these are acceptable, then optimality is an unambiguously good thing. In Part 2 of this chapter we look at the way markets allocate resources and commodities. To anticipate, we shall see that what can be claimed for markets is that, given ideal institutional arrangements and certain modes of behaviour, they achieve allocative efficiency. It cannot be claimed that, alone, markets, even given ideal institutional arrangements, achieve what might generally or reasonably be regarded as fair allocations. Before looking at the way markets allocate resources, we shall look at economists' attempts to devise criteria for evaluating alternative allocations that do not involve explicit reference to a social welfare function.

5.4 Compensation tests

If there were a generally agreed SWF, there would be no problem, in principle, in ranking alternative allocations. One would simply compute the value taken by the SWF for the allocations of interest, and rank by the computed values. An allocation with a higher SWF value would be ranked above one with a lower value. There is not, however, an agreed SWF. The relative weights to be assigned to the utilities of different individuals are an ethical matter. Economists prefer to avoid specifying the SWF if they can. Precisely the appeal of the Pareto improvement criterion – a reallocation is desirable if it increases somebody's utility without reducing anybody else's utility – is that it avoids the need to refer to the SWF to decide on whether or not to recommend that reallocation. However, there are two problems, at the level of principle, with this criterion. First, as we have seen, the recommendation that all reallocations satisfying this condition be undertaken does not fix a unique allocation. Second, in considering policy issues there will be very few proposed reallocations that do not involve some individuals gaining and some losing. It is only rarely, that is, that the welfare economist will be asked for advice about a reallocation that improves somebody's lot without damaging somebody else's. Most reallocations that require analysis involve winners and losers and are, therefore, outside of the terms of the Pareto improvement criterion.

Given this, welfare economists have tried to devise ways, which do not require the use of a SWF, of comparing allocations where there are winners and losers. These are compensation tests. The basic idea is simple. Suppose there are two allocations, denoted 1 and 2, to be compared. As previously, the essential ideas are covered if we consider a two-person, two-commodity world. Moving from allocation 1 to allocation 2 involves one individual gaining and the other losing. The Kaldor compensation test, named after its originator, Nicholas Kaldor, says that allocation 2 is superior to allocation 1 if the winner could compensate the loser and still be better off. Table 5.1 provides a numerical illustration of a situation where the Kaldor test has 2 superior to 1. In this, constructed, example, both individuals have

Table 5.1 Two tests, two answers

	Allocation 1			Allocation 2		
	X	*Y*	*U*	*X*	*Y*	*U*
A	10	5	50	20	5	100
B	5	20	100	5	10	50

Table 5.2 Two tests, one answer

	Allocation 1			Allocation 2		
	X	*Y*	*U*	*X*	*Y*	*U*
A	10	5	50	20	10	200
B	5	20	100	5	10	50

utility functions that are $U = XY$, and A is the winner for a move from 1 to 2, while B loses from such a move. According to the Kaldor test, 2 is superior because at 2 A could restore B to the level of utility that he enjoyed at 1 and still be better off than at 1. Starting from allocation 2, suppose that 5 units of *X* were shifted from A to B. This would increase B's utility to 100 (10×10), and reduce A's utility to 75 (15×5) – B would be as well off as at 1 and A would still be better off than at 1. Hence, the argument is: allocation 2 must be superior to 1, as, if such a reallocation were undertaken, the benefits as assessed by the winner would exceed the losses as assessed by the loser. Note carefully that this test does not require that the winner actually does compensate the loser. It requires only that the winner could compensate the loser, and still be better off. For this reason, the Kaldor test, and the others to be discussed below, are sometimes referred to as 'potential compensation tests'. If the loser was actually fully compensated by the winner, and the winner was still better off, then we would be looking at a situation where there was a Pareto improvement.

The numbers in Table 5.1 have been constructed so as to illustrate a problem with the Kaldor test. This is that it may sanction a move from one allocation to another, but that it may also sanction a move from the new allocation back to the original allocation. Put another way, the problem is that if we use the Kaldor test to ask whether 2 is superior to 1 we may get a 'yes', and we may also get a 'yes' if we ask if 1 is superior to 2. Starting from 2 and considering a move to 1, B is the winner and A is the loser. Looking at 1 in this way, we see that if 5 units of *Y* were transferred from B to A, B would have *U* equal to 75, higher than in 2, and A would have *U* equal to 100, the same as in 2. So, according to the Kaldor test done this way, 1 is superior to 2.

This problem with the Kaldor test was noted by J.R. Hicks, who actually put things in a slightly different way. He proposed a different (potential) compensation test for considering whether the move from 1 to 2 could be sanctioned. The question in the Hicks test is: could the loser compensate the winner for forgoing the move and be no worse off than if the move took place. If the answer is 'yes', the reallocation is not sanctioned, otherwise it is on this test. In Table 5.1, suppose at allocation 1 that 5 units of *Y* are transferred from B, the loser from a move to 2, to A. Now A's utility would then go up to 100 (10×10), the same as in allocation 2, while B's would go down to 75 (5×15), higher than in allocation 2. The loser in a reallocation from 1 to 2 could, that is, compensate the individual who would benefit from such a move for its not actually taking place, and still be better off than if the move had taken place. On this test, allocation 1 is superior to allocation 2.

In the example of Table 5.1, the Kaldor and Hicks (potential) compensation tests give different answers about the rankings of the two allocations under consideration. This will not be the case for all reallocations that might be considered. Table 5.2 is a, constructed, example where both tests give the same answer. For the Kaldor test, looking at 2, the winner A could give the loser B 5 units of *X* and still be better off than at 1 ($U = 150$), while B would then be fully compensated for the loss involved in going from 1 to 2 ($U = 10 \times 10 = 100$). On this test, 2 is superior to 1. For the Hicks test, looking at 1, the most that the loser B could transfer to the winner A so as not to be worse off than in allocation 2 is 10 units of *Y*. But, with 10 of *X* and 15 of *Y*, A would have $U = 150$, which is less than A's utility at 2, namely 200. The loser could not compensate the winner for forgoing the move and be no worse off than if the move took place, so again 2 is superior to 1.

For an unambiguous result from a (potential) compensation test, it is necessary to use both the Kaldor and the Hicks criteria. The Kaldor–Hicks–Scitovsky test – known as such because

Table 5.3 Compensation may not produce fairness

	Allocation 1			Allocation 2		
	X	*Y*	*U*	*X*	*Y*	*U*
A	10	5	50	10	4	40
B	5	20	100	15	16	240

Tibor Scitovsky pointed out that both criteria are required – says that a reallocation is desirable if:

(i) the winners could compensate the losers and still be better off
and
(ii) the losers could not compensate the winners for the reallocation not occurring and still be as well off as they would have been if it did occur.

In the example of Table 5.2 the move from 1 to 2 passes this test; in that of Table 5.1 it does not.

As we shall see, especially in Chapters 11 and 12 on cost–benefit analysis and environmental valuation respectively, compensation tests inform much of the application of welfare economics to environmental problems. Given that utility functions are not observable, the practical use of compensation tests does not take the form worked through here, of course. Rather, as we shall see, welfare economists work with monetary measures which are intended to measure utility changes. As noted above, the attraction of compensation tests is that they do not require reference to a SWF. However, while they do not require reference to a SWF, it is not the case that they solve the problem that the use of a SWF addresses. Rather, compensation tests simply ignore the problem. As indicated in the examples above, compensation tests treat winners and losers equally. No account is taken of the fairness of the distribution of well-being.

Consider the example in Table 5.3. Considering a move from 1 to 2, A is the loser and B is the winner. As regards (i), at 2 moving one unit of Y from B to A would make A as well off as she was at 1, and would leave B better off (U = 225) than at 1. As regards (ii), at 1 moving either two of X or one of Y from A to B would leave A as well off as at 2, but in neither case would this be sufficient to compensate B for being at 1 rather than 2 (for B after such transfers U = 140 or U = 105). According to both (i) and (ii) 2 is superior to 1, and such a reallocation passes the Kaldor–Hicks–Scitovsky test. Note, however, that A is the poorer of the two individuals, and that the reallocation sanctioned by the compensation test makes A worse off, and makes B better off. In sanctioning such a reallocation, the compensation test is either saying that fairness is irrelevant or there is an implicit SWF such that the reallocation is consistent with the notion of fairness that it embodies. If, for example, the SWF was

$$W = 0.5U^{A} + 0.5U^{B}$$

then at 1 welfare would be 75 and at 2 it would be 140. Weighting A's losses equally with B's gains means that 2 is superior to 1 in welfare terms. If it were thought appropriate to weight A's losses much more heavily than B's gains, given that A is relatively poor, then using, say

$$W = 0.95U^{A} + 0.05U^{B}$$

gives welfare at 1 as 52.5 and at 2 as 50, so that 1 is superior to 2 in welfare terms, notwithstanding that the move from 1 to 2 is sanctioned by the (potential) compensation test.

In the practical use of compensation tests in applied welfare economics, welfare, or distributional, issues are usually ignored. The monetary measures of winners' gains (benefits) and losers' losses (costs) are usually given equal weights irrespective of the income and wealth levels of those to whom they accrue. In part, this is because it is often difficult to identify winners and losers sufficiently closely to be able to say what their relative income and wealth levels are. But, even in those cases where it is clear that, say, costs fall mainly on the relatively poor and benefits mainly on the better off, economists are reluctant to apply welfare weights when applying a compensation test by comparing total gains and total losses – they simply report on whether or not £s of gain exceed £s of loss. Various justifications are offered for this practice. First, at the level of principle, that there is no generally agreed SWF for them to use, and it would be inappropriate for economists to themselves specify a SWF. Second, that, as a practical matter, it aids clear thinking to separate matters of efficiency from matters of equity, with the question of the relative sizes of gains and losses being treated as an efficiency issue, while the question of their incidence across poor and rich is an equity issue. On this view, when considering some policy intended to effect a reallocation the job of the economic analyst is to ascertain whether the gains exceed the losses. If they do, the policy can be recommended on efficiency grounds, and it is known that the beneficiaries could compensate the losers. It is a separate matter, for government, to decide whether compensation should actually occur, and to arrange for it to occur if it is thought desirable. These matters are usually considered in the context of a market economy, and we shall return to them in that context at the end of Part 2 of the chapter.

PART 2 ALLOCATION IN A MARKET ECONOMY

5.5 Efficiency given ideal conditions

A variety of institutional arrangements might be employed to allocate resources, such as dictatorship, central planning and free markets. Any of these can, in principle, achieve an efficient allocation of resources. Here, we are particularly interested in the consequences of free-market resource allocation decisions. This is for three, related, reasons. First, for dictatorship and central planning to achieve allocative efficiency it is necessary that the dictator or central planner know all of the economy's production and utility functions. This is clearly infeasible, and is one of the reasons that attempts to run economies in these ways have been unsuccessful. The great attraction of free markets as a way of organising economic activity is that they do not require that any institution or agent have such knowledge. That is the second reason for our concentration on markets – they are decentralised information-processing systems of great power. The third reason is that the modern welfare economics that is the basis for environmental and resource economics takes it that markets are the way economies are mainly organised. Environmental and resource issues are studied, that is, as they arise in an economy where markets are the dominant social institution for organising production and consumption. The market economy is now the dominant mode of organising production and consumption in human societies.

Welfare economics theory points to a set of circumstances such that a system of free markets would sustain an efficient allocation of resources. The 'institutional arrangements', as we shall call them, include the following:

1. Markets exist for all goods and services produced and consumed.
2. All markets are perfectly competitive.
3. All transactors have perfect information.
4. Private property rights are fully assigned in all resources and commodities.
5. No externalities exist.
6. All goods and services are private goods. That is, there are no public goods.
7. All utility and production functions are 'well behaved'.[3]

In addition to these institutional arrangements, it is necessary to assume that the actors in such a system – firms and individuals, often referred to jointly as 'economic agents' or just 'agents' – behave in certain ways. It is assumed that agents always strive to do the best for themselves that they can in the circumstances that they find themselves in. Firms are assumed to maximise profits, individuals to maximise utility. A shorthand way of saying this is to say that all agents are maximisers.

An efficient allocation would be the outcome in a market economy populated entirely by maximisers and where all of these institutional arrangements were in place. Before explaining why and how this is so, a few brief comments are in order on these conditions required for a market system to be capable of realising allocative efficiency. First, note that,

[3] For a full account of what 'well behaved' means the reader is referred to one of the welfare economics texts cited in the Further Reading section at the end of the chapter. Roughly, in regard to utility it means that indifference curves are continuous and have the bowed-toward-the-origin shape that they are usually drawn with in the textbooks. In regard to production, the main point is that increasing returns to scale are ruled out.

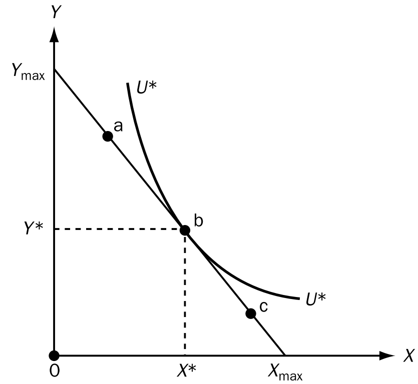

Figure 5.8 Utility maximisation

as we shall see in later sections of this chapter where we discuss public goods and externalities, arrangements 5 and 6 are really particulars of 4. Second, note that 4 is necessary for 1 – markets can only work where there are private property rights and a justice system to enforce and protect such rights. Third, that an important implication of 2 is that buyers and sellers act as 'price-takers', believing that the prices that they face cannot be influenced by their own behaviour. No agent, that is, acts in the belief that they have any power in the market. Finally, note that these are a very stringent set of conditions, which do not accurately describe any actual market economy. The economy that they do describe is an ideal type, to be used in the welfare analysis of actual economies as a benchmark against which to assess performance, and to be used to devise policies to improve the performance, in regard to efficiency criteria, of such actual economies.

We now explain why a market allocation of resources would be an efficient allocation in such ideal circumstances. A more formal treatment is provided in Appendix 5.2.

Consider, first, individuals and their consumption of produced commodities. Any one individual seeks to maximise utility given income and the, fixed, prices of commodities. Figure 5.8, familiar from introductory microeconomics, refers to an individual in a two-commodity economy. The line $Y_{max}X_{max}$ is the budget constraint. Y_{max} is the amount of Y available if all income is spent on Y, X_{max} is consumption if all income is spent on X. The slope of the budget constraint gives the price ratio P_X/P_Y. Utility maximisation requires consumption X^* and Y^* corresponding to point b on the indifference curve U^*U^*. Consumption at points on $Y_{max}X_{max}$ to the left or right of b, such as a and c, would mean being on a lower indifference curve than U^*U^*. Consumption patterns corresponding to points to the northeast of $Y_{max}X_{max}$ are not attainable with the given income and prices. The essential characteristic of b is that the budget line is tangential to an indifference curve. This means that the slope of the indifference curve is equal to the price ratio. Given that the slope of the indifference curve is the MRUS, we have:

$$\text{MRUS} = \frac{P_X}{P_Y}$$

In the ideal conditions under consideration, all individuals face the same prices. So, for the two-individual, two-commodity market economy, we have

$$\text{MRUS}^{\text{A}} = \text{MRUS}^{\text{B}} = \frac{P_X}{P_Y} \tag{5.8}$$

Comparison of equation 5.8 with equation 5.3 shows that the consumption efficiency condition is satisfied in this ideal market system. Clearly, the argument here generalises to many-person, multi-commodity contexts.

Now consider firms. To begin, instead of assuming that they maximise profits, we will assume that they minimise the costs of producing a given level of output. The cost-minimisation assumption is in no way in conflict with the assumption of profit maximisation. On the contrary, it is implied by the profit-maximisation assumption, as, clearly, a firm could not be maximising its profits if it were producing whatever level of output that involved at anything other than the lowest possible cost. We are leaving aside, for the moment, the question of the determination of the profit-maximising level of output, and focusing instead on the prior question of cost minimisation for a given level of output. This question is examined in Figure 5.9, where X^*X^* is the isoquant corresponding to some given output level X^*. The straight lines K_1L_1, K_2L_2, and K_3L_3 are isocost lines. For given prices for inputs, P_K and P_L, an isocost line shows the combinations of input levels for K and L that can be purchased for a given total expenditure on inputs. K_3L_3 represents, for example,

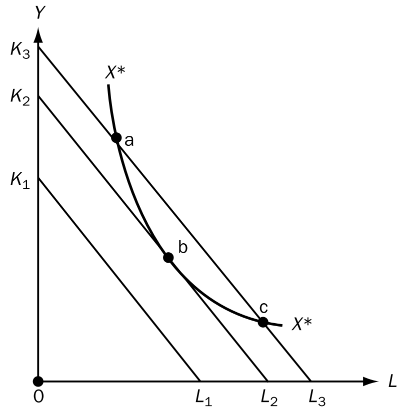

Figure 5.9 Cost minimisation

a higher level of expenditure on inputs, greater cost, than K_2L_2. The slope of an isocost line is the ratio of input prices, P_K/P_L. Given production of X^*, the cost-minimising firm will choose the input combination given by the point b. Any other combination, such as a or c, lying along X^*X^* would mean higher total costs. Combinations represented by points lying inside K_2L_2 would not permit of the production of X^*. The essential characteristic of b is that an isocost line is tangential to, has the same slope as, an isoquant. The slope of an isoquant is the MRTS so that cost-minimising choices of input levels must be characterised by:

$$\text{MRTS} = \frac{P_K}{P_L}$$

In the ideal circumstances under consideration, all firms, in all lines of production, face the same P_K and P_L, which means that

$$\text{MRTS}_X = \text{MRTS}_Y \tag{5.9}$$

which is the same as equation 5.4, the production efficiency condition for allocative efficiency – cost-minimising firms satisfy this condition.

The remaining condition that needs to be satisfied for allocative efficiency to exist is the product mix condition, equation 5.5, which involves both individuals and firms. In explaining how this condition is satisfied in an ideal market system we will also see how the profit-maximising levels of production are determined. Rather than look directly at the profit-maximising output choice, we look at the choice of input levels that gives maximum profit. Once the input levels are chosen, the output level follows from the production function. Consider the input of labour to the production of X, with marginal product X_L. Choosing the level of X_L to maximise profit involves balancing the gain from using an extra unit of labour against the cost of so doing. The gain here is just the marginal product of labour multiplied by the price of output, i.e. P_XX_L. The cost is the price of labour, i.e. P_L. If P_L is greater than P_XX_L, increasing labour use will reduce profit. If P_L is less than P_XX_L, increasing labour use will increase profit. Clearly, profit is maximised where $P_L = P_XX_L$.

The same argument applies to the capital input, and holds in both lines of production. Hence, profit maximisation will be characterised by

$$P_XX_L = P_L$$

$$P_XX_K = P_K$$

$$P_YY_L = P_L$$

$$P_YY_K = P_K$$

which imply

$$P_XX_L = P_YY_L = P_L$$

and

$$P_XX_K = P_YY_K = P_K$$

Using the left-hand equalities here, and rearranging, this is

$$\frac{P_X}{P_Y} = \frac{Y_L}{X_L} \tag{5.10a}$$

and

$$\frac{P_X}{P_Y} = \frac{Y_K}{X_K} \tag{5.10b}$$

Now, the right-hand sides here are MRT_L and MRT_K, as they are the ratios of marginal products in the two lines of production and hence give the terms on which the outputs change as labour and capital are shifted between industries. Given that the left-hand sides in equations 5.10a and 5.10b are the same we can write

$$\text{MRT}_L = \text{MRT}_K = \frac{P_X}{P_Y} \tag{5.11}$$

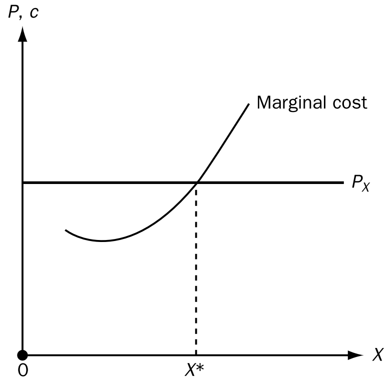

Figure 5.10 Profit maximisation

showing that the marginal rate of transformation is the same for labour shifting as for capital shifting. Referring back to equation 5.8, we can now write

$$\text{MRT}_L = \text{MRT}_K = \frac{P_X}{P_Y} = \text{MRUS}^{\text{A}} = \text{MRUS}^{\text{B}} \tag{5.12}$$

showing that the profit-maximising output levels in the ideal market economy satisfy the product mix condition for allocative efficiency, equation 5.5.

This completes the demonstration that in an ideal market system the conditions necessary for allocative efficiency will be satisfied. We conclude this section by looking briefly at profit-maximising behaviour from a perspective that will be familiar from an introductory microeconomics course. There, students learn that in order to maximise profit, a firm which is a price-taker will expand output up to the level at which price equals marginal cost. Figure 5.10 refers. For output levels below X^*, price exceeds marginal cost so that increasing output will add more to receipts than to costs, so increasing profit as the difference between receipts and costs. For output levels greater than X^*, marginal cost exceeds price, and reducing output would increase profit. This is in no way inconsistent with the discussion above of choosing input levels so as to maximise profit. It is just a different way of telling the same story. In order to increase output, assuming technical efficiency, more of at least one input must be used. In thinking about whether or not to increase output the firm considers increasing the input of capital or labour, in the manner described above. For the case of labour in the production of X, for example, the profit-maximising condition was seen to be $P_L = P_X X_L$, which can be written as

$$\frac{P_L}{X_L} = P_X$$

which is just marginal cost equals price, because the left-hand side is the price of an additional unit of labour divided by the amount of output produced by that additional unit. Thus if the wage rate is £5 per hour, and one hour's extra labour produces 1 tonne of output, the left-hand side here is £5 per tonne, so the marginal cost of expanding output by one tonne is £5. If the price that one tonne sells for is greater(less) than £5 it will pay in terms of profit to increase(decrease) output by one tonne by increasing the use of labour. If the equality holds and the output price is £5, profit is being maximised. The same argument goes through in the case of capital, and the marginal cost equals price condition for profit maximisation can also be written as

$$\frac{P_K}{X_K} = P_X$$

5.6 Partial equilibrium analysis of market efficiency

In examining the concepts of efficiency and optimality, we have used a general equilibrium approach. This looks at all sectors of the economy simultaneously. Even if we were only interested in one part of the economy – such as the production and consumption of cola drinks – the general equilibrium approach requires that we look at all sectors. In finding the allocatively efficient quantity of cola, for example, the solution we get from this kind of exercise would give us the efficient quantities of all goods, not just cola.

There are several very attractive properties of proceeding in this way. Perhaps the most important of these is the theoretical rigour it imposes. In developing economic theory, it is often best to use general equilibrium analysis. Much (although by no means all) of the huge body of theory that makes up resource and environmental economics analysis has such a general approach at its foundation.

But there are penalties to pay for this rigour. Doing applied work in this way can be expensive and time-consuming. And in some cases data limitations make it impossible. The exercise may not be quite as daunting as it sounds, however. We could define categories in such a way that there are just two goods in the economy: cola and a composite good that is everything except cola. Indeed, this kind of 'trick' is commonly used in economic analysis. But even with this type of simplification, a general equilibrium approach is likely to be difficult and costly, and may be out of all proportion to the demands of some problem for which we seek an approximate solution.

Given the cost and difficulty of using this approach for many practical purposes, many applications use a different framework that is much easier to operationalise. This involves looking at only the part of the economy of direct relevance to the problem being studied. Let us return to the cola example, in which our interest lies in trying to estimate the efficient amount of cola to be produced. The partial approach examines the production and consumption of cola, ignoring the rest of the economy. It begins by identifying the benefits and costs to society of using resources to make cola. Then, defining *net* benefit as total benefit minus total cost, an efficient output level of cola would be one that maximises net benefit.

Let X be the level of cola produced and consumed. Figure 5.11(a) shows the total benefits of cola (labelled B) and the total costs of cola (labelled C) for various possible levels of cola production. The reason we have labelled the curves $B(X)$ and $C(X)$, not just B and C, is to make it clear that benefits and costs each depend on, are functions of, X. Benefits and costs are measured in money units. The shapes and relative positions of the curves we have drawn for B and C are, of course, just stylised representations of what we expect them to look like. A researcher trying to answer the question we posed above would have to estimate the shapes and positions of these functions from whatever evidence is available, and they may differ from those drawn in the diagram. However, the reasoning that follows is not conditional on the particular shapes and positions that we have used, which are chosen mainly to make the exposition straightforward.

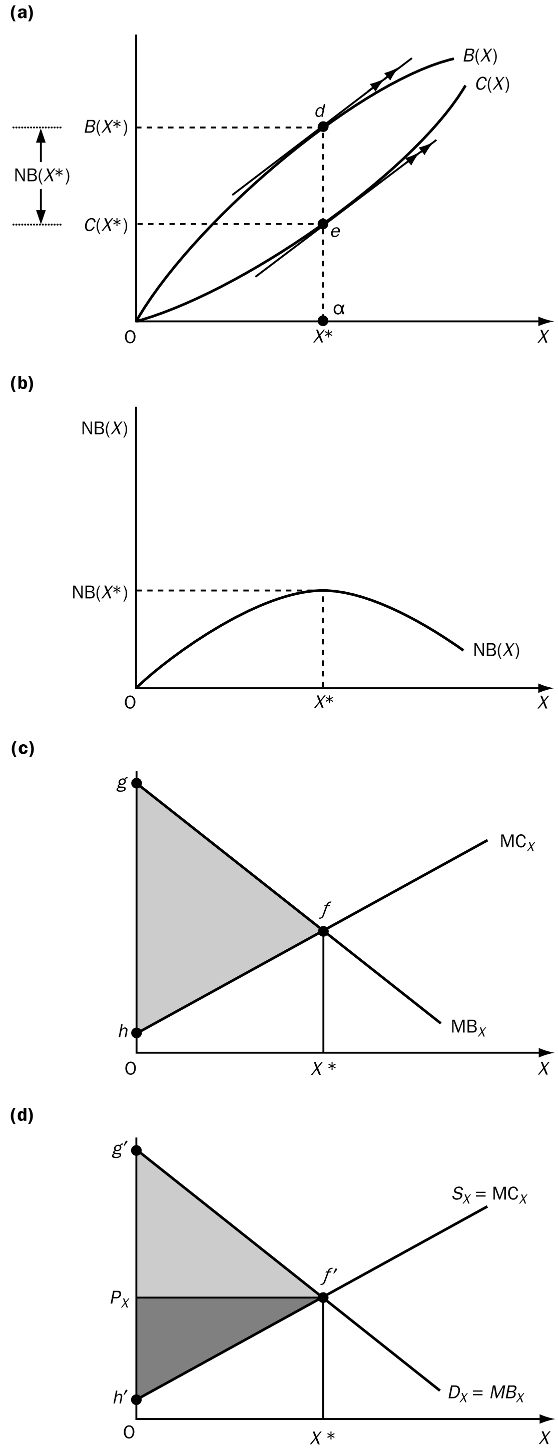

Figure 5.11 A partial equilibrium interpretation of economic efficiency

Given that we call an outcome that maximises net benefits 'efficient', it is clear from Figure 5.11(a) that X^* is the efficient level of cola production. Net benefits (indicated by the distance *de*) are at their maximum at that level of output. This is also shown in Figure 5.11(b), which plots the *net* benefits for various levels of X. Observe the following points:

- At the efficient output level X^* the total benefit and total cost curves are parallel to one another (Figure 5.11(a)).
- The net benefit function is horizontal at the efficient output level (Figure 5.11(b)).

The distance *de*, or equivalently the magnitude NB(X^*), where NB is net benefit, can be interpreted in efficiency terms. It is a measure, in money units, of the efficiency gain that would come about from producing X^* cola compared with a situation in which no cola was made.

These ideas are often expressed in a different, but exactly equivalent, way, using marginal rather than total functions. As much of the environmental economics literature uses this way of presenting ideas (and we shall do so also in several parts of this book), let us see how it is done. We use MC_X to denote the marginal cost of X, and MB_X denotes the marginal benefit of X. In Figure 5.11(c), we have drawn the marginal functions which correspond to the total functions in Figure 5.11(a). We drew the curves for $B(X)$ and $C(X)$ in Figure 5.11(a) so that the corresponding marginal functions are straight lines, a practice that is often adopted in partial equilibrium treatments of welfare economics. This is convenient and simplifies exposition of the subsequent analysis. But, the conclusions do not depend on the marginal functions being straight lines. The results to be stated hold so long as marginal benefits are positive and declining with X and marginal costs are positive and increasing with X – as they are in Figure 5.11(c).

In Figure 5.11(c) we show X^*, the cola output level that maximises net benefit, as being the level of X at which MC_X is equal to MB_X. Why is this so? Consider some level of X below X^*. This would involve MB_X greater than MC_X, from which it follows that increasing X would increase benefit by more than cost. Now consider some level of X greater than X^*, with MC_X greater than MB_X, from which it follows that reducing X would reduce cost by more than benefit, i.e. increase net benefit. Clearly, considering X levels above or below X^* in this way, it is X^* that maximises net benefit.

Can we obtain a measure of maximised net benefits from Figure 5.11(c) that corresponds to the distance *de* in Figure 5.11(a)? Such a measure is available; it is the area of the triangle *gfh*. The area beneath a marginal function over some range gives the value of the change in the total function for a move over that range. So the area beneath MB_X over the range $X = 0$ to $X = X^*$ gives us the total benefits of X^* cola (i.e. B^*), which is equal to the distance *ad* in Figure 5.11(a). Similarly, the area beneath MC_X over the range $X = 0$ to $X = X^*$ gives us the total cost of X^* (i.e. C^*), which is the same as the distance *ae* in Figure 5.11(a). By subtraction we find that the area *gfh* in Figure 5.11(c) is equal to the distance *de* in Figure 5.11(a).

Now we turn to the partial equilibrium version of the demonstration that an ideal market system maximises net benefit and secures allocative efficiency. We assume that all of the institutional arrangements listed in the previous section apply, and that all agents are maximisers. Then all those who wish to drink cola will obtain it from the market, and pay the going market price. The market demand curve, D_X, for cola will be identical to the MB_X curve, as that describes consumers' *willingness to pay* for additional units of the good – and that is exactly what we mean by a demand curve. Under our assumptions, cola is produced by a large number of price-taking firms in a competitive market. The market supply curve, S_X, is identical to the curve MC_X in Figure 5.11(c) because, given that firms produce where price equals marginal cost, the supply curve is just the marginal cost curve – each point on the supply curve is a point where price equals marginal cost. S_X shows the cost of producing additional (or marginal) cans of cola at various output levels.

The market demand and supply curves are drawn in Figure 5.11(d). When all mutually beneficial transactions have taken price, the equilibrium market price of the good will be P_X, equal at the margin to both

- consumers' subjective valuations of additional units of the good (expressed in money terms); and

- the costs of producing an additional unit of the good.

Put another way, all consumers face a common market price P_X, and each will adjust their consumption until their marginal utility (in money units) is equal to that price. Each firm faces that same fixed market price, and adjusts its output so that its marginal cost of production equals that price. So we have:

$$P_X = \mathrm{MC}_X = \mathrm{MB}_X \tag{5.13}$$

The equality at the margin of costs and benefits shows that cola is being produced in the amount consistent with the requirements of allocative efficiency. We must emphasise here something that it is sometimes possible to forget when using partial equilibrium analysis. The fact that equation 5.13 holds for the cola, or whatever, market means that the quantity of cola, or whatever, produced and consumed is consistent with allocative efficiency only if *all* the institutional arrangements listed at the start of this section are in place. It is necessary, for example, not only that the cola market be perfectly competitive, but also that all markets be perfectly competitive. And, it is necessary, for example, that all inputs to and outputs from production be traded in such markets. If such requirements are not met elsewhere in the economy, the supply and demand curves in the cola market will not properly reflect the costs and benefits associated with different levels of cola production. Some of the issues arising from these remarks will be dealt with in section 5.11 under the heading of 'the second-best problem'.

Finally here, we can use Figure 5.11(d) to introduce the concepts of *consumers' surplus* and *producers' surplus*, which are widely used in welfare economics and its application to environmental and natural resource issues. The area beneath the demand curve between zero and X^* units of the good shows the total consumers' willingness to pay, WTP, for X^* cans of cola per period. To see this, imagine a situation in which cans of cola are auctioned, one at a time. The price that the first can offered would fetch is given by the intercept of the demand curve, $0g'$. As successive cans are offered so the price that they fetch falls, as shown by the demand curve. If we add up all the prices paid until we get to X^*, and recognising that X^* is a very large number of cans, we see that the total revenue raised by the auction process which stops at X^* will be the area under the demand curve over $0X^*$, i.e. $0g'f'X^*$. But this is not the way the market works. Instead of each can being auctioned, a price is set and all cans of cola demanded are sold at that price. So, the individual who would have been willing to pay $0g'$ for a can actually gets it for P_X. Similarly, the individual who would have been willing to pay just a little less than $0g'$ actually pays P_X. And so on and so on, until we get to the individual whose WTP is P_X, and who also actually pays P_X. All individuals whose WTP is greater than P_X are, when all cans sell at P_X, getting a surplus which is the excess of their WTP over P_X. Consumers' surplus is the total of these individual surpluses, the area between the demand curve and the price line over $0X^*$, i.e. $P_Xg'f'$. Another way of putting this is that consumers' surplus is the difference between total willingness to pay and total actual expenditure, which is the difference between area $0g'f'X^*$ and area $0P_Xf'X^*$, which is the area of the triangle $P_Xg'f'$.

Producers' surplus in Figure 5.11(d) is the area of the triangle $h'P_Xf'$. The reasoning to this is very similar to that for consumers' surplus. As noted above, the supply curve is, given the ideal conditions being assumed here, just the marginal cost curve. The first can of cola costs $0h'$ to produce, but sells in the market for P_X, so there is a surplus of $h'P_X$. The surplus on the production of each further can is given by the vertical distance from the price line to the supply curve. The sum of all these vertical distances is total producers' surplus, the area $h'P_Xf'$. An alternative way of putting this is that total revenue is the area $0P_Xf'X^*$, while total cost is $0h'f'X^*$, so that producers' surplus is revenue minus costs, i.e. $h'P_Xf'X^*$.

5.7 Market allocations are not necessarily equitable

The previous sections have shown that, provided certain conditions are satisfied, a system of free markets will produce allocations that are efficient in the sense that nobody can be made better off except at the cost of making at least one other person worse off. It has not been shown that a system of free markets will produce an optimal allocation according to any particular social welfare function.

The basic intuition of both the positive – the attainment of efficiency – and the negative – no necessary attainment of equity – here is really rather simple. The essential characteristic of markets is voluntary exchange. Think of two individuals who meet, each carrying a box containing an assortment of commodities. The two assortments are different. The two individuals lay out the contents of their boxes, and swap items until there are no further swaps that both see as advantageous. Then, considering just these two individuals and the collection of commodities jointly involved, the allocation of that collection at the end of the swapping is efficient in the sense that if somebody else came along and forced them to make a further swap, one individual would feel better off but the other worse off, whereas prior to the enforced swap both felt better off than they did with their initial bundles. The attainment of efficiency is simply the exhaustion of the possibilities for mutually beneficial exchange. Clearly, if one individual's box had been several times as large as the other's, if one individual had a much larger initial endowment, we would not expect the voluntary trade process to lead to equal endowments. Voluntary trade on the basis of self-interest is not going to equalise wealth. Further, it is also clear that as the initial endowments of the two individuals – the sizes of their boxes and their contents – vary, so will the positions reached when all voluntary swaps have been made.

The formal foundations for modern welfare economics and its application to policy analysis in market economies are two fundamental theorems. These theorems take it that all agents are maximisers, and that the ideal institutional conditions stated at the start of this section hold. The first states that a competitive market equilibrium is an efficient allocation. Basically, this is saying that equilibrium is when there are no more voluntary exchanges, and that when there are no more voluntary exchanges all the gains from trade have been exhausted, so the situation must be one of efficiency – one where nobody can be made better off save at the cost of making somebody else worse off. The second theorem states that to every efficient allocation there corresponds a competitive market equilibrium based on a particular distribution of initial endowments. An alternative statement of this theorem, of particular relevance to policy analysis, is that any efficient allocation can be realised as a competitive market equilibrium given the appropriate set of lump-sum taxes on and transfers to individual agents. The point of the second theorem is that the efficient allocation realised by a competitive equilibrium is conditioned on the distribution of initial endowments, and that if those initial endowments are such that the resulting efficient allocation is considered inequitable, altering them by lump-sum taxes and transfers will produce another efficient allocation. If the taxes/transfers redistribute from the better to the worse off, the new efficient allocation will be more equitable.

The implication of these two theorems, which has enormous influence on the way that economists approach policy analysis in an economy mainly run by markets, is that there are two essentially separable dimensions to the economic problem. These are the problems of efficiency and equity. The theorems are taken to mean that, in effect, society can, via government, take a view on equity and achieve what it wants there by a system of redistributive taxes and payments, and then leave it to markets to achieve efficiency in allocation given the distribution of endowments after the tax/transfer. This can be put the other way round. The theorems are taken to mean that the government should not intervene in markets directly to pursue any equity objectives. It should not, for example, subsidise a commodity that figures largely in the consumption of the poor. To do so would prevent the market system attaining an efficient allocation. Anyway, it is unnecessary. The interests of the poor are to be looked after by redistributive taxes and transfers.

These theorems hold only in the ideal conditions being assumed in this part of the chapter. It will already have occurred to the reader that these conditions are not fully satisfied in any actual economy – we consider some violations and their policy implications in the next part of the chapter. It is also required that the government's redistribution be in the form of lump-sum taxes and transfers. By 'lump-sum' is meant taxes and transfers that do not directly affect the incentives facing agents – in the case of taxes, for example, liability must not depend on behaviour, so that income taxes are not lump-sum taxes. Lump-sum taxes and transfers are not, in fact, widely used by governments as they are generally seen as politically infeasible.

Notwithstanding that the conditions under which the two theorems hold are not fully satisfied in any actual economy, the overwhelming majority of economists do approach practical policy analysis on the basis that the problems of efficiency and equity can be dealt with independently.

PART 3 MARKET FAILURE, PUBLIC POLICY AND THE ENVIRONMENT

In Part 1 of this chapter, we laid out the conditions that characterise an efficient allocation. In Part 2, we showed that, given 'ideal' circumstances concerning institutions and behaviour, a system of markets would produce an efficient allocation. We noted that the ideal circumstances are truly ideal, in that they do not describe any actual economy. Actual market economies depart from the ideal circumstances in a variety of ways, and the allocations that they produce are not efficient. Economists use welfare economics to identify 'market failures' – situations where actual circumstances depart from the ideal – and to recommend policies to correct them so that actual economies perform better in relation to the objective of efficiency. Much of environmental and resource economics is welfare economics of this sort. It is concerned with identifying and correcting market failure in relation to the services that the environment provides to the economy. In this part of the chapter, we introduce some of the basic ideas involved here. In Part II of the book, we apply the basic ideas to the problem of environmental pollution. Part III extends the basic ideas to cover intertemporal allocation problems, and then looks, mainly, at the welfare economics of the amenity services that the environment provides. Part IV of the book then deals, mainly, with the economics of natural resources as inputs to production.

5.8 The existence of markets for environmental services

To recapitulate, we have seen that for markets to produce efficient allocations, it is necessary that:

1. Markets exist for all goods and services produced and consumed.
2. All markets are perfectly competitive.
3. All transactors have perfect information.
4. Private property rights are fully assigned in all resources and commodities.
5. No externalities exist.
6. All goods and services are private goods. That is, there are no public goods.
7. All utility and production functions are 'well behaved'.
8. All agents are maximisers.

Clearly, 1 here is fundamental. If there are goods and services for which markets do not exist, then the market system cannot produce an efficient allocation, as that concept applies to all goods and services that are of interest to any agent, either as utility or production function arguments. Further, 4 is necessary for 1 – a market in a resource or commodity can only exist where there are private property rights in that resource or commodity.

We can define a property right as: a bundle of characteristics that convey certain powers to the owner of the right.[4] These characteristics concern conditions of appropriability of returns, the ability to divide or transfer the right, the degree of exclusiveness of the right, and the duration and enforceability of the right. Where a right is exclusive to one person or corporation, a private property right is said to exist.

In Chapter 2 we provided a classification of the services that the natural environment provides to economic activity, using Figure 2.1. Let us now briefly consider the different classes of service distinguished there in relation to the question of the existence of private property rights. Where these do not exist, market forces cannot allocate efficiently. If efficiency is the objective, some kind of public

[4] This definition is taken from Hartwick and Olewiler (1986).

policy intervention is required. Our remarks here are intended only to provide a general overview, as a guide to what follows in the rest of this book. The details of any particular case can be quite complicated.

In regard to the provision of inputs to production, natural resources, we made two major distinctions – between flow and stock resources, and, for the latter, between renewables and non-renewables. Generally, there are no private property rights in flow resources as such. Individuals or corporations do not, for example, have property rights in flows of solar radiation. They may, however, have property rights in land, and, hence, in the ability to capture the solar radiation falling on that land.[5] Deposits of non-renewable natural resources are, generally, subject to private property rights. Often these reside ultimately with the government, but are sold or leased by it to individuals and/or corporations. The problems arising from the non-existence of private property rights are not central to the economics of non-renewable resources.

They do, on the other hand, feature large in the renewable resource economics literature. Many, but not all, of the biotic populations exploited by humans as hunter–gatherers, rather than agriculturists, are not subject to private property rights. The standard example of the case where they are not is the ocean fishery. Where private property rights are absent, two sorts of situation may obtain. In the case of 'open-access resources' exploitation is uncontrolled. The term 'common-property resources' is used whenever some legal or customary conventions, other than private property rights, regulate exploitation of the resource. Whereas an open-access regime definitely will not promote exploitation that corresponds to efficiency, a common property regime may do so given the appropriate conventions and regulation. Much of the modern fisheries economics literature, as will be seen in Chapter 17, is concerned with the design of systems of government regulation of common property that will promote behaviour consistent with efficiency on the part of the private agents actually exploiting the fishery.

The second class of environmental service that was distinguished was that of receptacle for the wastes arising in economic activity. Generally, for most of history and for many wastes, the environment as waste sink has not been subject to private property rights, and has been, in effect, an open-access resource. With increasing awareness of the problems of pollution arising, states have moved to legislate so as to convert many waste sinks from open-access resources to common-property resources. Much of Part II of the book is about the economic analysis that is relevant to the public-policy questions arising. What is the level of pollution that goes with efficiency? How should the behaviour of waste dischargers be regulated? We shall introduce the basic ideas involved here later in this chapter, when discussing 'externalities'.

The case of the amenity services that the environment provides is rather like that of flow resources, in that the service itself will not generally be subject to private property rights, though the means of accessing it may be. Thus, for example, nobody can own a beautiful view, but the land that it is necessary to visit in order to see it may be privately owned. Private property rights in a wilderness area would allow the owner to, say, develop it for agriculture or extractive resource use, thus reducing the amenity services flow from the area, or to preserve the wilderness. While in principle the owner could charge for access to a wilderness area, in practice this is often infeasible. Further, some of the amenity services that the area delivers do not require access, and cannot be charged for by the owner. The revenue stream that is available under the preservation option is likely to understate the true value to society of that option. This is not true of the development option. In this case, a decision as between the options based on market revenues will be biased in favour of the development option, and the operative question in terms of market failure is whether the existing private property rights need to be attenuated, so as to secure the proper, efficient, balance between preservation and development. This sort of issue is dealt with in Part III of the book.

[5] To see the complexities that can arise, note that in some jurisdictions a householder may be able to prevent others taking action which reduces the light reaching her property, though this may depend on the nature and purpose of the action.

The life-support services provided by the natural environment are not subject to private property rights. Consider, as an example, the global atmosphere, the carbon cycle and the climate system. Historically, the global atmosphere has been a free-access resource. As briefly discussed in Chapter 2, and to be revisited at several places in the rest of the book (especially Chapter 10), anthropogenic emissions of carbon dioxide have increased atmospheric concentrations of that greenhouse gas. The consensus of expert judgement is that this has affected the way that the global climate system works, and that unless action is taken to reduce the rate of growth of anthropogenic carbon dioxide emissions, further change, on balance harmful to human interests, will occur. Given this, most nations are now parties to an international agreement to act to curb the rate of growth of carbon dioxide and other greenhouse gas emissions. This agreement is discussed in Chapter 10. It can be seen as a first step in a process of transforming the global atmosphere from a free-access to a common-property resource.

5.9 Public goods

One of the circumstances, 6 in the listing above, required for it to be true that a pure market system could support an efficient allocation is that there be no public goods. Some of the services that the natural environment provides to economic activity have the characteristics of public goods, and cannot be handled properly by a pure market system of economic organisation. So we need to explain what public goods are, the problems that they give rise to for markets, and what can be done about these problems.

5.9.1 What are public goods?

This turns out to be a question to which there is no simple short answer. Public goods have been defined in different ways by different economists. At one time it was thought that there were just private goods and public goods. Now it is recognised that pure private and pure public goods are polar cases, and that a complete classification has to include intermediate cases. It turns out that thinking about these matters helps to clarify some other issues relevant to resource and environmental economics.

There are two characteristics of goods and services that are relevant to the public/private question. These are rivalry and excludability. What we call rivalry is sometimes referred to in the literature as divisibility. Table 5.4 shows the fourfold classification of goods and services that these two characteristics give rise to, and provides an example of each type. Rivalry refers to whether one agent's consumption is at the expense of another's consumption. Excludability refers to whether agents can be prevented from consuming. We use the term 'agent' here as public goods may be things that individuals consume and/or things that firms use as inputs to production. In what follows here we shall generally discuss public goods in terms of things that are of interest to individuals, and it should be kept in mind that similar considerations can arise with some inputs to production.

Pure private goods exhibit both rivalry and excludability. These are 'ordinary' goods and services, the example being ice cream. For a given amount of ice cream available, any increase in consumption by A must be at the expense of consumption by others, is rival. Any individual can be excluded from ice cream consumption. Ice cream comes in discrete units, for each of which a con-

Table 5.4 Characteristics of private and public goods

	Excludable	Non-excludable
Rivalrous	**Pure private good** Ice cream	**Open-access resource** Ocean fishery (outside territorial waters)
Non-rivalrous	**Congestible resource** Wilderness area	**Pure public good** Defence

sumption entitlement can be identified and traded (or gifted). Pure public goods exhibit neither rivalry nor excludability. The example given is the services of the national defence force. Whatever level that it is provided at is the same for all citizens of the nation. There are no discrete units, entitlement to which can be traded (or gifted). One citizen's consumption is not rival to, at the cost of, that of others, and no citizen can be excluded from consumption.

Open-access natural resources exhibit rivalry but not excludability. The example given is an ocean fishery that lies outside of the territorial waters of any nation. In that case, no fishing boat can be prevented from exploiting the fishery, since it is not subject to private property rights and there is no government that has the power to treat it as common property and regulate its exploitation. However, exploitation is definitely rivalrous. An increase in the catch by one fishing boat means that there is less for other boats to take.

Congestible resources exhibit excludability but not, up to the point at which congestion sets in, rivalry. The example given is the services to visitors provided by a wilderness area. If one person visits a wilderness area and consumes its services – recreation, wildlife experiences and solitude, for example – that does not prevent others from consuming those services as well. There is no rivalry between the consumption of different individuals, provided that the overall rate of usage is not beyond a threshold level at which congestion occurs in the sense that one individual's visit reduces another's enjoyment of theirs. In principle, excludability is possible if the area is either in private ownership or subject to common-property management. In practice, of course, enforcing excludability might be difficult, but, often, given limited points of access to vehicles it is not.

The question of excludability is a matter of law and convention, as well as physical characteristics. We have already noted that as the result of an international agreement that extended states' territorial waters, some ocean fisheries that were open access have become common property. We also noted above that a similar process may be beginning in respect to the global atmosphere, at least in regard to emissions into it of greenhouse gases. In some countries beaches cannot be privately owned, and in some such cases while beaches actually have the legal status of common property they are generally used on a free-access basis. This can lead to congestion. In other countries private ownership is the rule, and private owners do restrict access. In some cases where the law enables excludability, either on the basis of private ownership or common property, it is infeasible to enforce it. However, the feasibility of exclusion is a function of technology. The invention of barbed wire and its use in the grazing lands of North America is a historical example. Satellite surveillance could be used to monitor unauthorised use of wilderness areas, though clearly this would be expensive, and presumably at present it is not considered that the benefit from so doing is sufficient to warrant meeting the cost.

In the rest of this section we shall consider pure public goods, which we will refer to simply as 'public goods'. As noted, we will be returning to a detailed consideration of open-access resources, and common-property resources, at several places later in the book. Box 5.2 considers some examples of public goods. Box 5.3 looks at property rights in relation to biodiversity, and the arising implications for incentives regarding conservation and medicinal exploitation.

5.9.2 Public goods and economic efficiency

For our economy with two persons and two private goods, we found that the top-level, product-mix, condition for allocative efficiency was

$$MRUS^A = MRUS^B = MRT \tag{5.14}$$

which is equation 5.8 written slightly differently. As shown in Appendix 5.3, for a two-person economy where X is a public good and Y is a private good, the corresponding top-level condition is:

$$MRUS^A + MRUS^B = MRT \tag{5.15}$$

We have shown that, given certain circumstances, the first of these will be satisfied in a market economy. It follows that the condition which is equation 5.15 will not be satisfied in a market economy. A pure market economy cannot supply a public good at the level required by allocative efficiency criteria.

A simple numerical example can provide the rationale for the condition that is equation 5.15.

Box 5.2 Examples of public goods

The classic textbook examples of public goods are lighthouses and national defence systems. These both possess the properties of being non-excludable and non-rival. If you or I choose not to pay for defence or lighthouse services, we cannot be excluded from the benefits of the service, once it is provided to anyone. Moreover, our consumption of the service does not diminish the amount available to others. Bridges also share the property of being non-rival (provided they are not used beyond a point at which congestion effects begin), although they are not typically non-excludable.

Many environmental resources are public goods, as can be seen from the following examples. You should check, in each case, that the key criterion of non-rivalry is satisfied. The benefits from biological diversity, the services of wilderness resources, the climate regulation mechanisms of the earth's atmosphere, and the waste disposal and reprocessing services of environmental sinks all constitute public goods, provided the use made of them is not excessive. Indeed, much public policy towards such environmental resources can be interpreted in terms of regulations or incentives designed to prevent use breaking through such threshold levels.

Some naturally renewing resource systems also share public goods properties. Examples include water resource systems and the composition of the earth's atmosphere. Although in these cases consumption by one person does potentially reduce the amount of the resource available to others (so the resource could be 'scarce' in an economic sense), this will not be relevant in practice as long as consumption rates are low relative to the system's regenerative capacity.

Finally, note that many public health measures, including inoculation and vaccination against infectious diseases, have public goods characteristics, by reducing the probability of any person (whether or not he or she is inoculated or vaccinated) contracting the disease. Similarly, educational and research expenditures are, to some extent, public goods.

Box 5.3 Property rights and biodiversity

Among the many sources of value that humans derive from biological diversity is the contribution it makes to the pharmaceutical industry. This is examined in a volume which brings together a collection of papers on the theme of property rights and biological diversity (Swanson, 1995a). In this box we summarise some of the central issues raised there.

Swanson begins by noting that the biological characteristics of plants (and, to a lesser extent, animals) can be classified into primary and secondary forms. Primary characteristics concern the efficiency with which an organism directly draws upon its environment. For example, plant growth – and the survivability of a population of that plant over time – depends upon its rate of photosynthesis, by which solar energy is converted into the biological material of the plant itself. The success of a species depends on such primary characteristics; indeed, the ecological dominance of humans can be described largely in terms of the massive increases in primary productivity attained through modern agriculture.

But another set of characteristics – secondary characteristics – are also of great importance in the survivability of an organism within its environment. To survive in a particular ecological complex, an organism must be compatible with other living components of its environment. The secondary metabolites which plants develop are crucial in this respect. Some plants develop attractors (such as fruits and aromas) which increase the spread of their reproductive materials. Acorns, for example, are transported and eaten by small animals, thereby encouraging the spread of oak woodlands. Other plants develop repellents in the form of (unattractive) aromas or toxins, which give defence against predatory organisms.

A diverse ecosystem will be characterised by a large variety of biological organisms in which evolutionary processes generate a rich mix of these secondary metabolites. Many of these will be highly context-specific. That is, even within one fairly narrow class of plants, there can be a large variety of these secondary metabolites that function to give relative fitness in a particular

Box 5.3 ***continued***

location. These secondary characteristics are helpful to plants and animals not only in aiding current survival but also in terms of long-term evolutionary sustainability. The presence of a diverse collection of secondary metabolites provides resources to help organisms survive environmental disruptions.

But these secondary characteristics are also of immense value to humans, and have been for much of recorded history. Let us look at a few examples discussed by Swanson. Lemons have been used to avoid scurvy in humans for hundreds of years, without any knowledge about how this beneficial effect was taking place. We now know that the active ingredient is vitamin C, one of the secondary metabolites of citrus fruits. Similarly, the bark of the willow tree was used for pain relief for centuries before the active substance (salicylic acid) was identified; its current form is the drug aspirin. More recently, the plant sweetclover was found to be causing severe internal bleeding in cattle. Trials showed that it served as an anti-coagulant across a wide variety of animals. Subsequent developments led to its use in warfarin (the major rodent poison in the world) and in drugs to treat victims of strokes (to reduce blood clotting).

Until recently, almost all medicines were derived more or less directly from natural sources. Even today, in the modern pharmaceuticals industry, a large proportion of the drugs in use throughout the world are derived from natural sources. Much work within the pharmaceuticals industry is concerned with identifying medicinal uses of secondary metabolites within plant, animal and microbial communities. The first step in this process is to develop chemicals from these organisms that have demonstrable biological effects within humans. Possible uses of the chemicals can then be found. What is interesting is that even today, the drugs developed in this way (such as those used in general anaesthesia) are often used without good understanding of their mechanism.

Two things are virtually certain. First, a large number of substances are being, or have been, used in specific cultural contexts without their usefulness having become generally known. Secondly, we have only begun to scratch the surface of the range of possible uses that the biosphere permits. Our collective knowledge encompasses only a small part of what there is to know.

All of this suggests that the conservation of biological diversity is of enormous value. This was recognised in the 1992 Rio Convention on Biological Diversity, which stated that biological diversity must be conserved and cultural/institutional diversity respected. Yet the institutional arrangements we have in place are poorly designed to conserve that diversity.

Swanson focuses on the role that property rights plays. The nub of the problem is that the system of property rights which has been built up over the past 100 years rewards the creators of information in very different ways. Consider a drug company that extracts biological specimens from various parts of the world and screens these for potential beneficial effects. Intellectual property rights will be awarded to the first individual or organisation that can demonstrate a novel use of information in a product or process. There is nothing wrong with this, of course. A system which rewards people who create useful information by granting them exclusive rights to market products that incorporate that information is of immense value. Intellectual property rights, in the form of patents and the like, give market value to information, and create incentives to search for and exploit more information.

However, Swanson points out that not all forms of information have such market value. In particular, the existence of biologically diverse ecosystems creates a reservoir of potentially useful information, but no system of property rights exists which rewards those who build up or sustain biodiversity. He writes (1995a, p. 6):

> Internationally-recognised property rights systems must be flexible enough to recognise and reward the contributions to the pharmaceutical industry of each people, irrespective of the nature of the source of that contribution. In particular, if one society generates information useful in the pharmaceutical industry by means of investing in natural capital (non-conversion of forests etc.) whereas another generates such information by investing in human capital (laboratory-based research and school-based training) each is equally entitled to an institution that recognises that contribution.

What is needed, therefore, is a property rights system that brings the value of biodiversity back into human decision-making. So-called ‘intellectual’ property rights should be generalised to include not only intellectual but natural sources of information. Put another

Box 5.3 *continued*

way, it is *information* property rights rather than just *intellectual* property rights that should be protected and rewarded. An ideal system would reward any investment that generates information, including that which is produced naturally.

It is ironic that the 'success' of modern scientific systems of medicine may be contributing to a loss of potentially useful information. Swanson points to the fact that knowledge which is used with demonstrable success in particular cultural contexts often fails to be widely recognised and rewarded. The difficulty has to do with the fact that this knowledge is not codified in ways that satisfy conventional scientific standards. Publication in academic and professional journals, for example, tends to require analysis in a standard form of each link in the chain running from chemical input to accomplished objective. Unconventional or alternative forms of medicine that cannot fit this pattern struggle to survive, even when they have demonstrable value and where no orthodox substitute exists (such as in the treatment of eczema). Reading the collection of papers in full will show you what Swanson and his co-authors recommend to rectify these shortcomings.

Source: Swanson (1995a, b)

Suppose that an allocation exists such that MRT = 1, $MRUS^A = 1/5$ and $MRUS^B = 2/5$, so that $MRUS^A + MRUS^B < MRT$. The fact that the MRT is 1 means that, at the margin, the private and public commodities can be exchanged in production on a one-for-one basis – the marginal cost of an extra unit of X is a unit of Y, and vice versa. The fact that $MRUS^A$ is 1/5 means that A could suffer a loss of 1 unit of X, and still be as well off if she received 1/5th of a unit of Y by way of compensation. Similarly, the fact that $MRUS^B$ is 2/5 means that B could suffer a loss of 1 unit of X, and still be as well off if he received 2/5 of a unit of Y by way of compensation. Now, consider a reduction in the production of X by 1 unit. Since X is a public good, this means that the consumption of X by both A and B will fall by 1 unit. Given the MRT of 1, the resources released by this reduction in the production of X will produce an extra unit of Y. To remain as well off as initially, A requires 1/5 of a unit of Y and B requires 2/5 of a unit. The total compensation required for both to be as well off as they were initially is $1/5 + 2/5 = 3/5$ units of Y, whereas there is available 1 unit of Y. So, at least one of them could actually be made better off than initially, with neither being worse off. This would then be a Pareto improvement. Hence, the initial situation with $MRUS^A + MRUS^B < MRT$ could not have been Pareto optimal, efficient.

Now consider an initial allocation where MRT = 1, $MRUS^A = 2/5$ and $MRUS^B = 4/5$ so that $MRUS^A + MRUS^B > MRT$. Consider an increase of 1 unit in the supply of the public good, so that the consumption of X by both A and B increases by 1 unit. Given MRT = 1, the supply of Y falls by 1 unit. Given $MRUS^A = 2/5$, A could forgo 2/5 units of Y and remain as well off as initially, given X^A increased by 1. Given $MRUS^B = 4/5$, B could forgo 4/5 units of Y and remain as well off as initially, given X^B increased by 1. So, with an increase in the supply of X of 1 unit, the supply of Y could be reduced by $2/5 + 4/5 = 6/5$ without making either A or B worse off. But, in production the Y cost of an extra unit of X is just 1, which is less than 6/5. So, either A or B could actually be made better off using the 'surplus' Y. For $MRUS^A + MRUS^B > MRT$ there is the possibility of a Pareto improvement, so the initial allocation could not have been efficient.

Since both $MRUS^A + MRUS^B < MRT$ and $MRUS^A + MRUS^B > MRT$ are situations where Pareto improvements are possible, it follows that $MRUS^A + MRUS^B = MRT$ characterises situations where they are not, so it is a necessary condition for allocative efficiency.

In the case of a private good, each individual can consume a different amount. Efficiency requires, however, that all individuals must, at the margin, value it equally. It also requires, see equation 5.14, that the common valuation, at the margin, on the part of individuals is equal to the cost, at the margin, of the good. In the case of a public good, each individual must, by virtue of non-rivalry, consume the same amount of the good. Efficiency does not require that they all value it equally at the margin. It does require, see equation 5.15, that the sum of their marginal valuations be equal to the cost, at the margin, of the good.

Markets cannot provide public goods in the amounts that go with allocative efficiency. In fact, markets cannot supply public goods at all. This follows from their non-excludability characteristic. A market in widgets works on the basis that widget makers exchange the rights to exclusive control over defined bundles of widgets for the rights to exclusive control over defined bundles of something else. Usually, the exchange takes the form of the exchange of widgets for money. This can only work if the widget maker can deny access to widgets to those who do not pay, as is the case with private goods. Where access to widgets is not conditional on payment, a private firm cannot function as it cannot derive revenue from widget production. Given that the direct link between payment and access is broken by non-excludability, goods and services that have that characteristic have to be supplied by some entity that can get the revenue required to cover the costs of production from some source other than the sale of such goods and services. Such an entity is government, which has the power to levy taxes so as to raise revenue. The supply of public goods is (part of) the business of government. The existence of public goods is one of the reasons why all economists see a role for government in economic activity.

Given that it is the government that must supply a public good, the question which naturally arises for an economist is: what rule should government follow so as to supply it in amounts that correspond to efficiency? In principle, the answer to this question follows from equation 5.15. In a two-person, two-commodity economy, the efficient level of supply for the public good is the level at which the sum of two MRUSs is equal to the MRT between it and the private good. Actual economies have many individuals and many private commodities. The first point here presents no difficulty, as it is clear that we simply need to extend the summation over all MRUSs, however many there are. As regards the second, it is simply a matter of noting that the MRT is the marginal cost in terms of forgone private goods consumption, so that the rule becomes: supply the public good at the level where the sum of all the MRUSs is equal to the marginal cost. Now, it follows from its definition that the MRUS is the same as marginal willingness to pay, MWTP, so this rule can be stated as: supply the public good at the level where aggregate marginal willingness to pay is equal to marginal cost. The determination of the efficient amount of a public good, for two individuals for convenience, is illustrated in Figure 5.12.

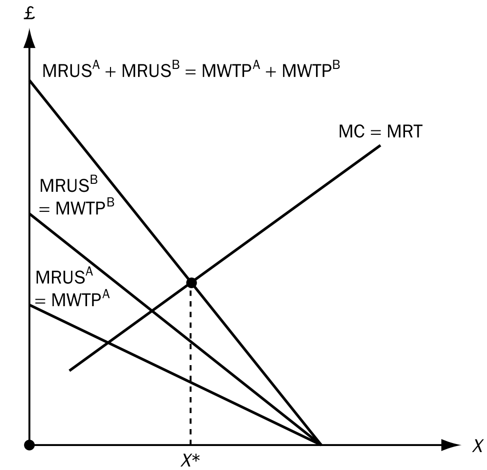

Figure 5.12 The efficient level of supply for a public good

5.9.3 Preference revelation and the free-rider problem

While the rule for the efficient supply of a public good is simple enough at the level of principle, its practical application faces a major difficulty. In order to apply the rule, the government needs to know the preferences, in terms of marginal willingness to pay, of all relevant individuals. It is in the nature of the case that those preferences are not revealed in markets. Further, if the government tries to find out what they are in some other way, then individuals have (on the standard assumptions about their motivations and behaviour) incentives not to truthfully reveal their preferences. Given that all consume equal amounts of a public good, and that exclusion from consumption on account of non-payment is impossible, individuals will try to 'free-ride' with respect to public goods provision.

To bring out the basic ideas here in a simple way we shall consider an example where the problem is to decide whether or not to provide a discrete amount of a public good, rather than to decide how much of a public good to supply. The nature of the problem is the same in either case, but is easier to state and understand in the 'yes/no' case than in the 'how much?' case. At issue is the question of whether or not to install street lighting. We will first look at this when there is no government. There are two individuals A and B. Both have an endowment of private goods worth £1000. Installing the street lighting will cost £100. The two individuals both have preferences such that they would be willing to pay £60 for the installation of street lighting. The analysis that follows is not dependent on the two individuals being equally well off and having the same preferences, that just makes the story easier to tell initially. An obvious modification of the rule derived for the efficient level of provision of a public good derived above for the 'yes/no' situation is that the decision should be 'yes' if the sum of individuals' willingness to pay is equal to or greater than the cost. In this case it is greater: £60 + £60 = £120.

Now, suppose that A and B agree to proceed in the following way. Each will independently write down on a piece of paper either 'Buy' or 'Don't buy'. If when the two pieces of paper are brought together, both have said 'Buy', they buy the street lighting jointly and share the cost equally. For two 'Don't buy' responses, the street lighting is not bought and installed. In the event of one 'Buy' and one 'Don't buy', the street lighting is bought and the individual who voted 'Buy' pays the entire cost. The four possible outcomes are shown in the cells of Table 5.5 in terms of the monetary valuations on the part of each individual, that of A to the left of the slash, that of B to the right.[6]

Table 5.5 The preference revelation problem

		B	
		Buy	Don't buy
A	Buy	1010/1010	960/1060
	Don't buy	1060/960	1000/1000

In the bottom right cell, the decision is not to go ahead. Neither incurs any cost in regard to street lighting and neither gets any benefit, so both are in their initial situations with £1000. Suppose both responded 'Buy'. Then with the street lighting installed, as shown in the top left cell, the situation for both can be expressed in monetary terms as £1010. Each has paid £50, half of the total of £100, for something valued at £60, so gaining by £10 as compared with the no street lighting situation. Suppose A wrote 'Buy' and B wrote 'Don't buy'. The lighting goes in, A pays the whole cost and B pays nothing. A pays £100 for something she values at £60, and goes from £1000 to £960. B pays nothing for something he values at £60, and goes from £1000 to £1060. This is shown in the top right cell. The bottom left cell has the entries of that cell reversed, because B pays the whole cost.

Now, clearly both are better off if both write 'Buy' and the street lighting is bought. But, either will be even better off if, as in the bottom left or top right cell, they can 'free-ride'. For each individual thinking about what to write on their piece of paper, writing 'Don't buy' is the dominant strategy. Consider individual B. If A goes for 'Buy', B gets to £1010 for 'Buy' and to £1060 for 'Don't buy'. If A goes for 'Don't buy', B gets to £960 for 'Buy' and to £1000 for 'Don't buy'. Whatever A does, B is better off going for 'Don't buy'. And the same is true for A, as can readily be checked. So, while installing the lighting and sharing the cost equally is a Pareto improvement, it will not come about where both

[6] This is a 'game' with the structure often referred to as 'the prisoner's dilemma' because of the setting in which the structure is often articulated. A 'game' is a situation in which agents have to take decisions the consequences of which depend on the decisions of other agents. We shall come back to looking at some game structures in Chapter 10. In the prisoner's dilemma setting, the agents are two individuals arrested for a crime and subsequently kept apart so that they cannot communicate with one another. The evidence against them is weak, and the police offer each a deal – confess to the crime and get a much lighter sentence than if you are convicted without confessing. Confession by one implicates the other. If neither confesses both go free. If both confess, both get lighter sentences. If only one confesses, the confessor gets a light sentence while the other gets a heavy sentence. The dominant strategy is confession, though both would be better off not confessing.

individuals act independently to serve their own self-interest. What is needed is some kind of coordination, so as to bring about the Pareto improvement which is going ahead with the street lighting.

Given what we have already said about public goods, government would seem the obvious way to bring about the required coordination. It can, in principle, ascertain whether the installation of street lighting is justified on efficiency grounds, and if it is install it and cover the cost by taxing each individual according to their willingness to pay. However, in practice, given self-seeking individuals, the free-rider problem also attends this programme. The problem comes up in trying to get the individuals to reveal their true preferences for the public good.

Suppose now that a government does exist, and that it wants to follow efficiency criteria. It knows that installing the street lighting will cost £100, and that it should install it if total willingness to pay is equal to or greater than that. It does not know the preferences, in terms of willingness to pay, of the two individuals who, in this simple example, constitute the citizenry. The obvious thing for it to do is to ask them about it. It does that, stating that the cost of installation will be met by a tax on each individual which is proportional to their willingness to pay and such that the total tax raised is equal to the cost of installation. If each individual truly reports willingness to pay £60, the street lighting will go ahead and each will pay £50 in tax. This represents a Pareto improvement – see the top left cell in Table 5.5. The problem is that the incentives facing each individual are not such as to guarantee truthful preference revelation. Given that tax liability will be proportional to stated willingness to pay, there is an incentive to understate it so as to reduce the tax liability if the street lighting goes ahead, and to get something of a free ride. In the example of Table 5.5, if B states willingness to pay as £40 and A tells the truth, the street lighting will go ahead – stated aggregate willingness to pay £100 – and B will pay 40%, rather than 50%, of £100. If A also understates willingness to pay by £20, the government's estimate of aggregate willingness to pay will mean that it does not go ahead with the lighting. The attempt to free-ride may fail if many make it.

The problem of securing truthful preference revelation in regard to the supply of public goods has been the subject of a lot of investigation by economists. It turns out to be very difficult to come up with systems that provide the incentives for truthful revelation, and are feasible. The interested reader will find references to work in this area in the Further Reading section at the end of the chapter. Here we will, in order to indicate the nature of the difficulties, simply note one idea that is intended to overcome the free-riding incentives generated by the system just discussed. There the problem was that an individual's tax liability depended on stated willingness to pay. This could be avoided by the government's asking about willingness to pay on the understanding that each individual would, if the installation went ahead, pay a fixed sum. Suppose that the government divided the cost by the number of individuals, and stated that the fixed sum was £50 per individual. For both individuals, true willingness to pay is £60. Both have an incentive now to overstate their willingness to pay. Both value the street lighting at more than it is going to cost them so they want to see it installed. Both know that this is more likely the higher they say that their willingness to pay is, and that however much in excess of £60 they report they will only pay £50.

In this case overstating willingness to pay produces the right decision. The street lighting should be installed on the basis of true aggregate willingness to pay, and will be installed on the basis of reported willingness to pay. If the lighting is installed, each individual is better off, there is a Pareto improvement. Suppose, however, that A's willingness to pay is £55 and B's is £40. In that case, aggregate willingness to pay is £95, less than the cost of £100, and the street lighting should not be installed. In this case, on the understanding that each would pay a tax of £50 if the lighting is installed, A would have an incentive to overstate her willingness to pay as before, but B would have an incentive to understate his. In fact, it would make sense for B to report willingness to pay as £0 – if the lighting goes ahead he pays £50 for something worth just £40 to him, so he will want to do the most he can to stop it going ahead. Whether it does go ahead or not depends on how much A overstates her willingness to pay by. If A reports £200 or more, despite B reporting £0, the street lighting will be installed when on efficiency grounds it should not be.

Finally, this simple example can be used to show that even if the government could secure the truthful revelation of preferences, public goods supply is still a difficult problem. Suppose that A's true willingness to pay is £60 and B's is £41, and that somehow or other the government knows this without needing to ask the individuals. The government has to decide how to cover the cost. It could tax each in proportion to willingness to pay, but given that A and B are initially equally wealthy in terms of private goods, this is in practice unlikely as it would be regarded as unfair. Taxing each at equal shares of the cost would be likely to be seen as the 'fair' thing to do. In that case, A would pay £50 for a benefit worth £60, and B would pay £50 for a benefit worth £41. In monetary terms, as the result of installing the lighting, A would go from £1000 to £1010 and B would go from £1000 to £991. Since there is a loser this is not a Pareto improvement, though it is a potential Pareto improvement – we are into the domain of the Kaldor–Hicks–Scitovsky test. By looking at equally wealthy individuals, we avoided the problem that efficiency gains are not necessarily welfare gains. Suppose that the gainer A were much richer than the loser B. Then, the question arises as to whether gains and losses should be given equal weight in coming to a decision.

For a government to make decisions about the supply and financing of public goods according to the criteria recommended by economists requires that it have lots of difficult-to-acquire information, and can involve equity questions as well as efficiency questions.

5.10 Externalities

An external effect, or an externality, is said to occur when the production or consumption decisions of one agent have an impact on the utility or profit of another agent in an unintended way, and when no compensation/payment is made by the generator of the impact to the affected party.[7] In our analysis thus far in this chapter, we have excluded the existence of externalities by the assumptions that were made about the utility and production functions. But in practice consumption and production behaviour by some agents does affect, in uncompensated/unpaid-for ways, the utility gained by other consumers and the output produced, and profit realised, by other producers. Economic behaviour does, in fact, involve external effects.

The stated definition of an external effect is not perhaps very illuminating as to what exactly is involved. Things will become clearer as we work through the analysis. The two key things to keep in mind are that we are interested in effects from one agent to another which are unintended, and where there is no compensation, in respect of a harmful effect, or payment, in respect of a beneficial effect. We begin our analysis of externalities by discussing the forms that externalities can take.

5.10.1 Classification of externalities

In our two-person, two-(private)-commodity, two-input economy we have worked with

$$U^{\mathrm{A}} = U^{\mathrm{A}}(X^{\mathrm{A}}, Y^{\mathrm{A}})$$

$$U^{\mathrm{B}} = U^{\mathrm{B}}(X^{\mathrm{B}}, Y^{\mathrm{B}})$$

as utility functions, and

$$X = X(K^X, L^X)$$

$$Y = Y(K^Y, L^Y)$$

as production functions. Note that here the only things that affect an individual's utility are her own consumption levels, and that the only things that affect a firm's output are the levels of inputs that it uses. There are, that is, no external effects.

[7] Some authors leave out from the definition of an externality the condition that the effect is not paid or compensated for, on the grounds that if there were payment or compensation then there would be no lack of intention involved, so that the lack of compensation/payment part of the definition as given in the text here is redundant. As we shall see, there is something in this. However, we prefer the definition given here as it calls attention to the fact that lack of compensation/payment is a key feature of externality as a policy problem. Policy solutions to externality problems always involve introducing some kind of compensation/payment so as to remove the unintentionality, though it has to be said that the compensation/payment does not necessarily go to/come from the affected agent.

Table 5.6 Externality classification

Arising in	Affecting	Utility/production function
Consumption	Consumption	$U^{A}(X^{A}, Y^{A}, X^{B})$
Consumption	Production	$X(K^{X}, L^{X}, Y^{A})$
Consumption	Consumption and production	$U^{A}(X^{A}, Y^{A}, X^{B})$ and $Y(K^{Y}, L^{Y}, X^{B})$
Production	Consumption	$U^{A}(X^{A}, Y^{A}, X)$
Production	Production	$X(K^{X}, L^{X}, Y)$
Production	Consumption and production	$U^{A}(X^{A}, Y^{A}, Y)$ and $X(K^{X}, L^{X}, Y)$

External effects can, first, be classified according to what sort of economic activity they originate in and what sort of economic activity they impact on. Given two sorts of economic activity, consumption and production, this gives rise to the sixfold classification shown in Table 5.6. The first column shows whether the originating agent is a consumer or producer, the second whether the affected agent is a consumer or producer, and the third provides an illustrative utility or production function for the affected agent. In Table 5.6, we are concerned only to set out the forms that unintended interdependence between agents could take. Some examples will be provided shortly.

In the first row in Table 5.6, an example of a consumption externality is where agent B's consumption of commodity X is an argument in A's utility function – B's consumption of X affects the utility that A derives from given levels of consumption of X and Y. In the second row, A's consumption of Y is shown as affecting the production of X, for given levels of capital and labour input. Row 3 has B's consumption of X affecting both A's utility and the production of Y. In row 4, the amount of X produced, as well as A's consumption of X, affects A's utility. Row 5 has the production of Y determining, for given capital and labour inputs, the amount of X produced. Finally, in row 6 we have a situation where the level of Y affects both A's utility and the production of X.

The unintended impact that an external effect involves may be harmful or beneficial. Table 5.7 provides examples of both kinds. If an individual has a vaccination that protects them, which is their intention, it also has the unintended effect of reducing the probability that others will contract the disease. An individual playing their radio loudly in the park inflicts suffering on others, though that is not their intention. In these two cases, the external effect originates in consumption and affects individuals. A beneficial externality originating in production, and impacting on production, is the case where a honey producer's bees pollinate a nearby fruit orchard. Pollution, in the bottom right cell, is a harmful externality which most usually originates in production activities. It can affect consumers, or producers, or both.

Another dimension according to which external effects can be classified is in terms of whether they have, or do not have, the public goods characteristics of non-rivalry and non-excludability. While external effects can have the characteristics of private goods, those that are most relevant for policy analysis exhibit non-rivalry and non-excludability. This is especially the case with external effects that involve the natural environment, which mainly involve pollution problems. Why this is the case will become clear in the analysis that follows here. All of the examples in Table 5.7 involve non-rivalry and non-excludability.

5.10.2 Externalities and economic efficiency

Externalities are a source of market failure. Given that all of the other institutional conditions for a pure market system to realise an efficient allocation hold, if there is a beneficial externality the market will produce too little of it in relation to the

Table 5.7 Beneficial and harmful externalities

Effect on others	Originating in consumption	Originating in production
Beneficial	Vaccination against an infectious disease	Pollination of blossom arising from proximity to apiary
Adverse	Noise pollution from radio playing in park	Chemical factory discharge of contaminated water into water systems

Box 5.4 Atmospheric ozone and market failure

Evidence now suggests that the accumulation of tropospheric ozone in urban areas poses serious threats to human health, and also leads to agricultural crop damage in surrounding areas.[8] A major source of tropospheric ozone is road vehicle exhaust emissions. Because vehicle emissions have real effects on well-being through our utility and production functions, these emissions can be termed 'goods' (although it may be preferable to label them as 'bads' as the effects on utility are adverse). However, with no individual private property rights in clean air, in the absence of government intervention, no charge is made for such emissions. With no charges being made for damaging emissions, resources will not be allocated efficiently. An efficient allocation would involve lower exhaust emissions, implying one or more of: lower traffic volumes, change in fuel type used, increased engine efficiency, enhanced exhaust control. How such objectives might be achieved is considered in this chapter, and in more detail in Chapter 7, but it should be clear at this stage that one method would be through the use of a tax on the emissions that cause ozone accumulation. An efficient emissions tax would impose a tax rate equal to the value of the marginal damage that would occur at the efficient level of emissions.

In arriving at this conclusion, we do not explicitly consider the time dimension of pollution. But note that if ozone accumulates over time, and damage is dependent on the stock of ozone rather than the flow of emissions in any particular period, then we need to consider the accumulation of the pollutant over time. As Chapter 16 shows, where emission flows lead to accumulating stocks of pollutants, it may be efficient to impose a tax rate that rises over time.

[8] Note that this accumulation of ozone in lower layers of the atmosphere is completely distinct from the destruction of the ozone layer in the earth's upper atmosphere (the stratosphere). The latter phenomenon – often known as 'holes in the ozone layer' – causes different problems, as is explained in Chapter 10.

requirements of allocative efficiency, while in the case of a harmful externality the market will produce more of it than efficiency requires. Since we are concerned with the application of welfare economics to environmental problems, and the main relevance of externalities there is in regard to environmental pollution, we shall look in any detail only at harmful externalities here. Box 5.4 concerns an important example of a harmful externality pollution problem. We will demonstrate that the market, in the absence of corrective policy, will 'over-supply' pollution by looking at three sorts of pollution problem – a consumer-to-consumer case, a producer-to-producer case, and a case where the unintended effect is from a producer to consumers. These three cases bring out all of the essential features of pollution as a market failure problem. In the text we shall use diagrams and partial equilibrium analysis to make the essential points – the reader may find it useful to review our exposition of this method of analysis in an earlier part of this chapter. In Appendix 5.3 we cover the same ground using general equilibrium analysis.

Before getting into these cases in a little detail, we can make a general intuitive point that covers both beneficial and harmful externalities. The basic problem with external effects follows directly from the definition in regard to unintendedness and lack of payment/compensation. These two features of the externality problem are directly related. The lack of intentionality follows from the fact that the impact involved does not carry with it any recompense, in the case of a beneficial effect, or penalty, in the case of a harmful effect. External effects arise where an agent's actions affecting other agents do not involve any feedback – benefit is conferred which is not rewarded, or harm is done which is not punished. Given the lack of reward/punishment, which in a market system would be signalled by monetary payment, an agent will not take any account of the effect concerned. It will be unintended and 'external' to their decision making. Where it is a beneficial effect, it will not be encouraged sufficiently, and there will not be enough of it. Where it is a harmful effect, it will not be discouraged sufficiently, and there will be too much of it. The key to dealing with the market failure that external effects give rise to is to put in place the missing feedbacks, to create a system which does reward/punish the generation of beneficial/harmful effects, so that they are no longer unintentional.

5.10.3 Consumption–consumption externality

Suppose that A and B live in adjacent flats (apartments). A is a saxophone player, who enjoys practising a lot. B does not like music, and can hear A practising. The utility functions are

$$U^A = U^A(M^A, S^A)$$

$$U^B = U^B(M^B, S^A)$$

where M represents wealth and S^A is the hours that A plays the saxophone each week, with $\partial U^A/\partial M^A > 0$, $\partial U^B/\partial M^B > 0$, $\partial U^A/\partial S^A > 0$ and $\partial U^B/\partial S^A < 0$. In Figure 5.13 we show, as MB, the marginal benefit of playing to A, and, as MEC for marginal external cost, the marginal cost of playing to B. Marginal benefit is the amount that A would pay, if it were necessary, to play a little more. Conversely, MB is the amount of compensation that would be required to leave A as well off given a small reduction in playing. Marginal external cost is the amount that B would be willing to pay for a little less playing. Conversely, MEC is the amount of compensation that would be required to leave B as well off given a small increase in M (hours of A's saxophone playing).

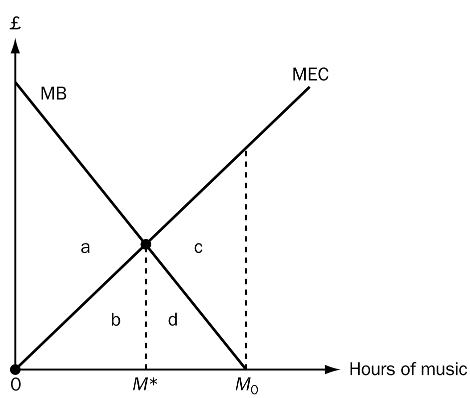

Figure 5.13 The bargaining solution to an externality problem

Given that A does not in fact have to pay anything to play her saxophone in her flat, she will increase her hours of playing up to the level M_0, where MB is equal to zero. At that level, A's total benefit from playing is given by the sum of the areas of the triangles a, b and d, and B's total suffering is measured in money terms by the sum of the areas b, d and c.

This is not an efficient outcome, because at M_0, MEC > MB. The efficient outcome is at M^* where MEC = MB. At any M to the left of M^*, MB > MEC, so that for a small increase in M, A would be willing to pay more than would compensate B for that increase. At any M to the right of M^*, MEC > MB so that for a small decrease in M, B would be willing to pay more for a small decrease in M than would be required to compensate A for that decrease. The inefficient level of saxophone playing at M_0 comes about because there are no payments in respect of variations in M, no market in M, so that the effect on B is unintentional on the part of A.

At the level of principle, the solution to this problem of inefficiency is fairly obvious. The problem is that A does not compensate B because B does not have any legal right to such compensation, does not have a property right in a domestic environment unpolluted by saxophone music. So, the solution is to establish such a property right, to give B the legal right to a domestic environment that is not noise-polluted. Such legal arrangements would support bargaining which would lead to M^* as the level of M. The argument that establishes that M^* would be the outcome under a legal regime where B can claim compensation from A exactly parallels the argument that establishes that M^* is the efficient outcome. To the left of M^*, with MB > MEC, A will be willing to pay more in compensation for a small increase in M than B requires, so will pay and play more. A will not increase M beyond M^* because the compensation that it would be necessary to pay B would be greater than the worth to A of the small increase thereby attained.

5.10.3.1 The Coase theorem

The idea that, given a suitable assignment of property rights, private bargaining between individuals can correct externality problems and lead to efficient outcomes is generally attributed to the Nobel prize winning economist Ronald Coase, and the result discussed above is often referred to as the 'Coase theorem' (the seminal paper is Coase, 1960). In fact, the result discussed above is only half of the Coase theorem. The other half says that an efficient outcome can also be attained by vesting the property right in the generator of the external effect. In that case, the generator would have the legal right to play, for this example, as much saxophone as she liked. The point is that given that right, it could be in the interests of the victim to offer money to the generator not to exercise their right to the full. Just as the absence of a clear property right vested in the victim inhibits one kind of bargaining, so does the absence of a clear property right vested in the generator inhibit another kind of bargaining.

Suppose then, that in our saxophone-playing example a law is passed saying that all saxophone players have an absolute right to practise up to the limits of their physical endurance. Legally A can play as much as she wants. But, a legal right can be traded. So, the opportunity now exists for A and B to bargain to a contract specifying the amount that A will actually play. That amount will be M^* in Figure 5.13. To the right of M^*, MEC > MB, so B's willingness to pay for a small reduction is greater than the compensation that A requires for that small reduction. Starting at M_0 and considering successive small reductions, B will be offering more than A requires until M^* is reached where B's offer will exactly match the least that A would accept. A and B would not be able to agree on a level of M to the left of M^*, since there B's willingness to pay is less than A requires by way of compensation.

So, what the Coase theorem actually says is that given this kind of externality situation, due to incomplete private property rights, one solution involves creating property rights for either the victim or the generator, and that either assignment will lead to an efficient outcome. It needs to be explicitly and carefully noted here that there are two things that are not being claimed. First, that it is not being said that the outcome will be the same in both cases. Second, that it is not being said that either way of assigning property rights necessarily promotes equity.

In regard to the first point here, note that considering the move from M_0 to M^* in our saxophone music example consequent upon the establishment of the property right and the ensuing bargaining we have:

(a) For the case where B gets the property right – there is an M reduction of $(M_0 - M^*)$ and A pays B an amount equal to the area of triangle b, the money value of B's suffering at the efficient outcome M^*.
(b) For the case where A gets the property right – there is an M reduction of $(M_0 - M^*)$ and B pays A an amount equal to the area of triangle d, the money value of A's loss as compared with the no-property-rights situation.

Clearly, which way the property right is assigned affects the wealth of A or B. To be granted a new property right is to have one's potential monetary wealth increased. In case (a), B experiences less saxophone hours and an increase in wealth by virtue of a payment from A, so that A's wealth goes down with her pleasure from playing. In case (b), B experiences less saxophone hours and a decrease in wealth by virtue of a payment to A, who gets less pleasure from playing. As we have drawn Figure 5.13, in neither (a) nor (b) does the increase in wealth affect the receiving individual's tastes. In case (a), that is, B's willingness to pay for less music hours is not affected by becoming wealthier – the slope of the MEC line does not change. In case (b), A's willingness to pay for more music hours is not affected by becoming wealthier – the slope and position of the MB line do not change. While these assumptions may be plausible in this example, they clearly are not generally appropriate. They were imposed here to produce a simple and clear graphical representation. If the assumption that tastes are unaffected by wealth increases is dropped, then with the case (a) assignment MEC would shift and with the case (b) assignment MB would shift. In neither case then would M^* as shown in Figure 5.13 be the bargaining outcome, and the outcomes would be different in the two cases. Both outcomes would be efficient, because in both cases we would have MB = MEC, but they would involve different levels of M.

So, the first point is that the Coase theorem properly understood says that there will be an efficient outcome under either assignment of property rights, not that there will be the same efficient outcome under either assignment. The second point, concerning equity, is simply that there is no implication that either assignment will have any desirable implications in terms of equity. This follows directly from our earlier discussions of the relationship between optimality and efficiency. In the case of our saxophone example, we have said nothing about the initial wealth/income situations of the two individuals. Clearly, our views on which way the property right should be assigned will, unless we are totally uninterested in equity, be affected by the wealth/income of the two individuals. Given that efficiency criteria do not discriminate between the two possible assignments of property rights, it might seem natural to take the view that the assignment should be on the basis of equity considerations. Unfortunately, this does not lead to any generally applicable rules. It is not always the case that externality sufferers are relatively poor and generators relatively rich, or vice versa. Even if we confine attention to a particular class of nuisance, such as saxophone playing in flats, it cannot be presumed that sufferers deserve, on equity grounds, to get the property right – some may be poor in relation to their neighbour and some rich.

Given the simple and compelling logic of the arguments of the Coase theorem, the question arises as to why uncorrected externalities are a problem. If they exist by virtue of poorly defined property rights and can be solved by the assignment of clearly defined property rights, why have legislatures not acted to deal with externality problems by assigning property rights? A full answer to this question would be well beyond the scope of this book, but the following points are worthy of note. First, as we have seen, the case for property rights solutions is entirely an efficiency case. Legislators do not give efficiency criteria the weight that economists do – they are interested in all sorts of other criteria. Second, even given clearly defined property rights, bargaining is costly. The costs increase with the number of participants. While expositions of the Coase theorem deal with small numbers of generators and sufferers, typically one of each, externality problems that are matters for serious policy concern generally involve many generators and/or many sufferers, and are often such that it is difficult and expensive to relate one particular agent's suffering to another particular agent's action. This makes bargaining expensive, even if the necessary property rights exist in law. The costs of bargaining, or more generally 'transactions costs', may be so great as to make bargaining infeasible. Third, even leaving aside the large numbers problem, in many cases of interest the externality has public bad characteristics which preclude bargaining as a solution.[9] We shall discuss this last point in the context of producer-to-consumer externalities.

5.10.4 Production–production externality

For situations where numbers are small, this case can be dealt with rather quickly. Consider two firms with production functions

$$X = X(K^X, L^X, S)$$

$$Y = Y(K^Y, L^Y, S)$$

where S stands for pollutant emissions arising in the production of Y, which emissions affect the output of X for given levels of K and L input there. As an example, Y is paper produced in a mill which discharges effluent S into a river upstream from a laundry which extracts water from the river to produce clean linen, X. Then, the assumption is that $\partial Y/\partial S > 0$, so that for given levels of K^Y and L^Y lower S emissions means lower Y output, and that $\partial X/\partial S < 0$, so that for given levels of K^X and L^X higher S means lower X.[10]

This externality situation is amenable to exactly the same kind of treatment as the consumer-to-consumer case just considered. Property rights could be assigned to the downstream sufferer or to the

[9] 'Public bad' is a term often used for a public good that confers negative, rather than positive, utility on those who consume it.

[10] Note that we are guilty here of something that we cautioned against in Chapter 2 in our discussion of the materials balance principle – writing a production function in which there is a material output, S, for which no material input basis is given. We do this in the interests of simplicity. A more appropriate production function specification is given in Appendix 5.3, where it is shown that the essential point for present purposes is not affected by our short-cut in the interests of simplicity.

upstream generator. Bargaining could then, in either case, produce an efficient outcome. To see this simply requires the reinterpretation and relabelling of the horizontal axis in Figure 5.13 so that it refers to S, with S_0 replacing M_0 and S^* replacing M^*. For profits in the production of X we have

$$\pi^X = P_X X(K^X, L^X, S) - P_K K^X - P_L L^X$$

where $\partial\pi^X/\partial S < 0$. The impact of a small increase in S on profits in the production of X is, in the terminology of Figure 5.13, marginal external cost, MEC. For profits in the production of Y we have

$$\pi^Y = P_X Y(K^Y, L^Y, S) - P_K K^Y - P_L L^Y$$

where $\partial\pi^Y/\partial S > 0$. The impact of a small increase in S on profits in the production of Y is, in the terminology of Figure 5.13, marginal benefit, MB. With these reinterpretations, the previous analysis using Figure 5.13 applies to the producer-to-producer case – in the absence of a well-defined property right S will be too large for efficiency, while an efficient outcome can result from bargaining based on a property right assigned to either the producer of X or the producer of Y.

An alternative way of internalising the externality would be to have the firms collude so as to maximise their joint profits. That this would produce an efficient outcome is proved in Appendix 5.3. The matter is, however, quite intuitive. The externality arises because the Y producer does not take account of the effects of its actions on the output for given inputs of the X producer. If the Y producer chooses its levels of K^Y, L^Y and S in the light of the consequences for the output of X for given K^X and L^X, and hence on the profits arising in the production of X, then those consequences will not be unintended. On the contrary, the two firms will be operated as if they were a single firm producing two commodities. We know that a single firm producing a single commodity will behave as required for efficiency, given all of the ideal conditions. All that is being said now is that this result carries over to a firm producing two commodities. For the firm that is producing both X and Y the ideal conditions do apply, as there is no impact on its activities the level of which is unintentionally set by others.

While joint profit maximisation can internalise an externality as required for efficiency, there appear to be few, if any, recorded instances of firms colluding, or merging, so as to internalise a pollution externality. Collusion to maximise joint profits will only occur if both firms believe that their share of maximised joint profits will be larger than the profits earned separately. There is, in general, no reason to suppose that cases where there is the prospect of both firms making higher profits with collusion will coincide with circumstances where there is a recognised inter-firm pollution externality.

5.10.5 Production–consumption externality

The key feature of the case to be considered now is that the external effect impact on two agents, and with respect to them is non-rival and non-excludable in consumption. As is the case generally in this chapter, 'two' is a convenient way of looking at 'many' – the two case brings out all the essential features of the many case while simplifying the notation and the analysis. Putting this key feature in the context of the production-to-consumption case aligns with the perceived nature of the pollution problems seen as most relevant to policy determination. These are typically seen as being situations where emissions arising in production adversely affect individuals in ways that are non-rival and non-excludable.

So, in terms of our two-person, two-commodity economy we assume that:

$$U^A = U^A(X^A, Y^A, S) \text{ with } \partial U^A/\partial S < 0$$

$$U^B = U^B(X^B, Y^B, S) \text{ with } \partial U^B/\partial S < 0$$

$$X = X(K^X, L^X)$$

$$Y = Y(K^Y, L^Y, S) \text{ with } \partial Y/\partial S > 0$$

Emissions arise in the production of Y and adversely affect the utilities of A and B. The pollution experienced by A and B is non-rival and non-excludable. A concrete example, bearing in mind that 'two' stands for 'many', would be a fossil-fuel-burning electricity plant located in an urban area. Its emissions pollute the urban airshed, and, to a first

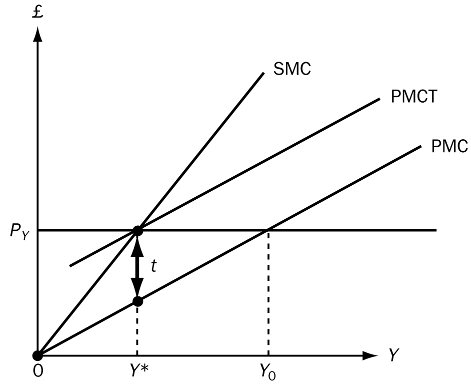

Figure 5.14 Taxation for externality correction

approximation, all who live within the affected area experience the same level of atmospheric pollution.

Given our earlier discussion of the supply of public goods, we can immediately conclude here that private bargaining based on some assignment of property rights will not deal with the externality problem. And, the joint profit maximisation solution is not relevant. In this kind of situation, correcting the market failure requires some kind of ongoing intervention in the workings of the market by some government agency. As we shall consider at some length and in some detail in Part II of the book, there is a range of means of intervention that the government agency, call it an environmental protection agency or EPA, could use. Here, we shall just look at the use of taxation by the EPA, so as to bring out the essential features of the situation where the externality has the characteristics of a public bad. A formal general equilibrium analysis is sketched in Appendix 5.3. Here we shall use partial equilibrium analysis based on Figure 5.14.

It introduces some new terminology. PMC stands for private marginal cost. Private costs are the input costs that the Y producer actually takes account of in determining its profit-maximising output level, i.e.

$$C = P_K K^Y + P_L L^Y = C(Y)$$

so that PMC = $\partial C/\partial Y$. We introduced the idea of MEC (marginal external cost) in considering the consumer-to-consumer case, as the amount that the sufferer would be willing to pay to reduce suffering by a small amount. In the present case there are two sufferers and MEC is the sum of the willingness to pay of each of them, as consumption of suffering is non-rival and non-excludable. We define social marginal cost as:

$$\text{SMC} = \text{PMC} + \text{MEC}$$

Figure 5.14 shows PMC increasing with Y in the usual way. The SMC line has a steeper slope than the PMC line, so that MEC is increasing with Y – as Y production increases, S output increases.

To maximise profit, the Y firm will produce at Y_0, where PMC is equal to the output price P_Y. This is not the Y output that goes with efficiency, as in balancing costs and benefits at the margin it is ignoring the costs borne by A and B. Efficiency requires the balancing at the margin of benefits and costs which include the external costs borne by A and B. The efficient output level for Y is, that is, Y^* where SMC equals P_Y. In the absence of any correction of the market failure that is the external costs imposed on A and B, the market-determined level of Y output will be too high for efficiency, as will the corresponding level of S.

To correct this market failure the EPA can tax S at a suitable rate. In Figure 5.14, we show a line labelled PMCT, which stands for private marginal cost with the tax in place. This line shows how the Y firm's marginal costs behave given that the EPA is taxing S at the appropriate rate. As shown in Figure 5.14, the appropriate tax rate is

$$t = \text{SMC}^* - \text{PMC}^* = \text{MEC}^* \tag{5.16}$$

that is, the tax needs to be equal to marginal external cost at the efficient levels of Y and S. In Appendix 5.3 we show that another way of stating this is:

$$t = P_X \lfloor \text{MRUS}^{\text{A}}_{XS} + \text{MRUS}^{\text{B}}_{XS} \rfloor \tag{5.17}$$

Comparing equations 5.16 and 5.17, we are saying that

$$\text{MEC}^* = P_X \lfloor \text{MRUS}^{\text{A}}_{XS} + \text{MRUS}^{\text{B}}_{XS} \rfloor \tag{5.18}$$

This makes a lot of sense. Recall that MRUS stands for marginal rate of utility substitution. The XS subscripts indicate that it is the MRUS for commodity X and pollution S that is involved here. Recall also that the MRUS gives the amount of the increase in, in this case, X that would keep utility constant in the face of a small increase in S. Equation 5.18 says that

MEC* is the monetary value of the extra consumption of commodity X by A and B that would be required to compensate them both for a small increase in S, from the efficient level of S. In saying this we are choosing to use the commodity X as the compensation vehicle. We could equally well have chosen the commodity Y for this purpose and derived

$$t = P_Y\lfloor \text{MRUS}^A_{YS} + \text{MRUS}^B_{YS}\rfloor \tag{5.19a}$$

and

$$\text{MEC}^* = P_Y\lfloor \text{MRUS}^A_{YS} + \text{MRUS}^B_{YS}\rfloor \tag{5.19b}$$

Taxation at the rate MEC* is required to bring about efficiency. Note that the tax rate required is not MEC at Y_0, is not MEC in the uncorrected situation. In order to be able to impose taxation of emissions at the required rate, the EPA would need to be able to identify Y^*. Given that prior to EPA intervention what is actually happening is Y_0, identification of Y^* and calculation of the corresponding MEC* would require that the EPA knew how MEC varied with S, i.e. knew the utility functions of A and B. It is in the nature of the case that this information is not revealed in markets. The problems of preference revelation in regard to public goods were discussed above. Clearly, those problems carry over to public bads such as pollution. The implications of this for feasible policy in respect of pollution control by taxation are discussed in Part II of the book.

Finally here we should note that the basic nature of the result derived here for the case where just one production activity gives rise to the emissions of concern carries over to the case where the emissions arise in more than one production activity. Consider a two-person, two-commodity economy where

$$U^A = U^A(X^A, Y^A, S) \text{ with } \partial U^A/\partial S < 0$$

$$U^B = U^B(X^B, Y^B, S) \text{ with } \partial U^B/\partial S < 0$$

$$X = X(K^X, L^X, S^X) \text{ with } \partial S/\partial S^X > 0$$

$$Y = Y(K^Y, L^Y, S^Y) \text{ with } \partial Y/\partial S^Y > 0$$

$$S = S^X + S^Y$$

Both production activities involve emissions of S, and both individuals are adversely affected by the total amount of S emissions. In this case, efficiency requires that emissions from both sources be taxed at the same rate, $t = \text{MEC}^*$.

5.11 The second-best problem

In our discussion of market failure thus far we have assumed that just one of the ideal conditions required for markets to achieve efficiency is not satisfied. Comparing our list of the institutional arrangements required for markets to achieve efficiency with the characteristics of actual economies indicates that the latter typically depart from the former in several ways rather than just in one way. In discussing harmful externalities generated by firms, we have, for example, assumed that the firms concerned sell their outputs into perfectly competitive markets, are price-takers. In fact, very few of the industries in a modern economy are made up of firms that act as price-takers.

An important result in welfare economics is the second-best theorem. This demonstrates that if there are two or more sources of market failure, correcting just one of them as indicated by the analysis of it as if it were the only source of market failure will not necessarily improve matters in efficiency terms. It may make things worse. What is required is an analysis that takes account of multiple sources of market failure, and of the fact that not all of them can be corrected, and derives, as 'the second-best policy', a package of government interventions that do the best that can be done given that not all sources of market failure can be corrected.

To show what is involved, we consider in Figure 5.15 an extreme case of the problem mentioned above, where the polluting firm is a monopolist. As

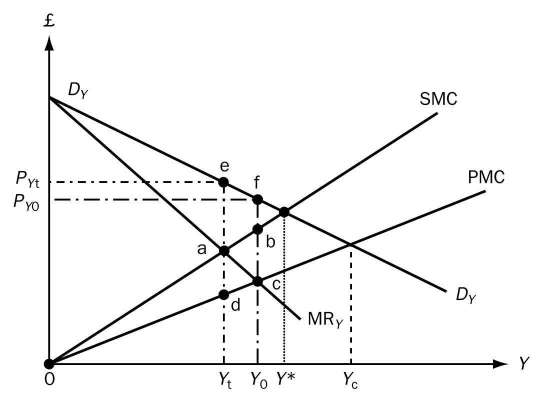

Figure 5.15 The polluting monopolist

above, we assume that the pollution arises in the production of Y. The profit-maximising monopolist faces a downward-sloping demand function, $D_Y D_Y$, and produces at the level where marginal cost equals marginal revenue, MR_Y. Given an uncorrected externality, the monopolist will use PMC here, and the corresponding output level will be Y_0. From the point of view of efficiency, there are two problems about the output level Y_0. It is too low on account of the monopolist setting marginal cost equal to marginal revenue rather than price: Y_c is the output level that goes with PMC = P_Y. It is too high on account of the monopolist ignoring the external costs generated and working with PMC rather than SMC: Y_t is the output level that goes with SMC = MR_Y. What efficiency requires is SMC = P_Y, with corresponding output level Y^*.

Now suppose that there is an EPA empowered to tax firms' emissions and that it does this so that for this monopolist producer of Y, SMC becomes the marginal cost on which it bases its decisions. As a result of the EPA action, Y output will go from Y_0 down to Y_t, with the price of Y increasing from P_{Y0} to P_{Yt}. The imposition of the tax gives rise to gains and losses. As intended, there is a gain in so far as pollution damage is reduced – the monetary value of this reduction is given by the area abcd in Figure 5.15. However, as a result of the price increase, there is a loss of consumers' surplus, given by the area P_{Yt}efP_{Y0}. It cannot be presumed generally that the gain will be larger than the loss. The outcome depends on the slopes and positions of PMC, SMC and $D_Y D_Y$, and in any particular case the EPA would have to have all that information in order to figure out whether imposing the tax would involve a net gain or a net loss.

When dealing with polluting firms that face downward-sloping demand functions, in order to secure efficiency in allocation the EPA needs two instruments – one to internalise the externality and another to correct under-production due to the firms' setting MC = MR rather than MC = P. With two such instruments, the EPA could induce the firm to operate at Y^* where SMC = P_Y. However, EPAs are not given the kinds of powers that this would require. They can tax emissions, but they cannot regulate monopoly. It can be shown that, given complete information on the cost and demand functions, and on how damages vary with the firm's behaviour, the EPA could figure out a second-best tax rate to be levied on emissions.[11] The second-best tax rate is one that guarantees that the gains from its imposition will exceed the losses. It does not move the firm to Y^* in Figure 5.15, but it does guarantee that the equivalent to abcd that it induces will be larger than the corresponding equivalent to P_{Yt}efP_{Y0}. The level of the second-best tax rate depends on the damage done by the pollutant, the firm's costs, and the elasticity of demand for its output. With many polluting monopolies to deal with, the EPA would be looking at imposing different tax rates on each, even where all produce the same emissions, on account of the different elasticities of demand that they would face in their output markets. It needs to be noted that charging different firms different rates of tax on emissions of the same stuff is unlikely to be politically feasible, even if the EPA had the information required to calculate the different rates.

5.12 Imperfect information

Given that all of the other ideal institutional arrangements are in place, the attainment of efficient outcomes through unregulated market behaviour presupposes that all transactors are perfectly informed about the implications for themselves of any possible transaction. This is clearly a strong requirement, not always satisfied in actual market economies. The requirement carries over to the analysis of the correction of market failure. Consider, to illustrate the point here, a case of consumption-to-consumption external effect where two individuals share a flat and where A is a smoker but B is not. Suppose that B does not find cigarette smoke unpleasant, and is unaware of the dangers of passive smoking. Then, notwithstanding that the government has legislated for property rights in domestic air unpolluted with cigarette smoke, B will not seek to reduce A's

[11] See Chapter 6 of Baumol and Oates (1988).

smoking. Given B's ignorance, the fact that bargaining is possible is irrelevant. The level of smoke that B endures will be higher than it would be if B were not ignorant. Given that B does not, when legally he could, bargain down A's level of smoking, we could describe the situation as one of 'conditional efficiency'. But this is not really very helpful. Rather, we recognise B's ignorance and consider it to be the source of an uncorrected externality. The nature of the corrective policy in the case of imperfect information is clear – the provision of information. In many cases, the information involved will have the characteristics of a public good, and there is a role for government in the provision of accurate information.

In some cases the government cannot fulfil this role because it does not have accurate and unambiguous information. Particularly where it is the future consequences of current actions that are at issue – as for example in the case of global warming – it may be simply impossible for anybody to have complete and accurate information. We all, as they say, live in an uncertain world. Imperfect information about the future consequences of current actions becomes particularly important in circumstances where those actions have irreversible consequences. It does appear to be the case that many of the consequences of decisions about environmental resource use are irreversible. Global warming may be a case in point. Again, it is arguable that, once developed, a natural wilderness area cannot be returned to its natural state. We take up some of the issues arising from such considerations in Parts III and IV of the book.

5.13 Government failure

We have shown that government intervention offers the possibility of realising efficiency gains, by eliminating or mitigating situations of market failure. First, many environmental resources are not subject to well-defined and clearly established property rights. As we have seen, efficiency gains may be obtained if government can create and maintain appropriate institutional arrangements for establishing and supporting property rights as the basis for bargaining. However, we have also seen that the scope of this kind of government action to correct market failure is limited to cases where non-rivalry and non-excludability are absent. Many environmental problems do involve non-rivalry and non-excludability. In such cases, possible government interventions to correct market failure are often classified into two groups. So-called command-and-control instruments take the form of rules and regulations prohibiting, limiting or requiring certain forms of behaviour. Fiscal instruments – tax and subsidy systems, and marketable permits – are designed to create appropriate patterns of incentives on private behaviour. We have looked at taxation briefly in this chapter, and we shall explore all of these instruments in depth in Chapter 7. As noted immediately above, another form that government intervention to correct market failure could take is providing information, or funding research activity that can increase the stock of knowledge. The arguments we have used so far in this chapter have all pointed to the possibility of efficiency gains arising from public-sector intervention in the economy. But actual government intervention does not always or necessarily realise such gains, and may entail losses. It would be wrong to conclude from an analysis of 'market failure' that all government intervention in the functioning of a market economy is either desirable or effective.

First, the removal of one cause of market failure does not necessarily result in a more efficient allocation of resources if there remain other sources of market failure. We discussed this above, using the case of the polluting monopolist as an illustration. A second consideration is that government intervention may itself induce economic inefficiency. Poorly designed tax and subsidy schemes, for example, may distort the allocation of resources in unintended ways. Any such distortions need to be offset against the intended efficiency gains when the worth of intervention is being assessed.

In some cases, the chosen policy instruments may simply fail to achieve desired outcomes. This is particularly likely in the case of instruments that take the form of quantity controls or direct regulation. One example of this is the attempt by the Greek government to reduce car usage, and hence congestion and pollution, in Athens. Regulations prohibiting entry into the city by cars with particular letters on their licence plates on particular days has served to promote the purchase of additional cars by households wishing to maintain freedom of mobility in the city. Similarly, the use of quantity controls in fisheries policy (such as determining minimum mesh sizes for nets, maximum number of days of permitted fishing, required days in port for vessels, and so on), intended to address the free-access problem of overexploitation, have met with very little success. Fishermen have responded to the regulations by making behavioural adjustments to minimise their impact. The limited success of quantitative controls in fishing is explored at length in Chapter 17.

It is not the case that actual government interventions are always motivated by efficiency, or even equity, considerations. It has been argued that the way government actually works in democracies can best be understood by applying to the political process the assumption of self-interested behaviour that economists use in analysing market processes. Four classes of political agent are distinguished: voters, elected members of the legislature, workers in the bureaucracy, and pressure groups. Voters are assumed to vote for candidates they believe will serve their own interests. Legislators are assumed to maximise their chances of re-election. Bureaucrats are assumed to seek to enlarge the size of the bureaucracy, so improving their own career prospects. Pressure groups push special interests with politicians and bureaucrats. The argument is that, given these motivations and circumstances, the outcome is not going to be a set of enacted policies that promote either efficiency or equity.

Politicians lack accurate information about voters' preferences. Voters lack reliable information about politicians' intentions. It is relatively easy for pressure groups to get their message across to politicians precisely because they focus on particular concerns arising from the strongly held views of a relatively small number of individuals or firms. Pressure groups access politicians directly, and via the bureaucracy. Bureaucrats, given their self-interest, amplify for politicians the messages from pressure groups that appear to call for a larger bureaucracy. They also control the flow of technical information to the politicians. The outcome of all this is, it is argued, an excessively large government doing, largely, things which keep, at least some, pressure groups happy, rather than things that reflect the preferences of the majority of voters.

Summary

In this chapter, we have defined and explained the terms 'efficiency' and 'optimality' as they are used in welfare economics. We have also demonstrated that a perfectly functioning 'ideal' market economy would bring about an efficient outcome, but not necessarily an optimal one.

However, it is clear that economies in practice do not satisfy the conditions of the ideal competitive economy that we described above. Markets are incomplete – there are many things that concern economic agents that are not traded in markets. Where they exist, markets are often not perfectly competitive. Many producers and consumers operate with information that is not perfect. Government must exist and raise revenue for the supply of public goods. Often, consumption and production behaviour generates uncompensated external effects upon others. These 'failures' will result in inefficient allocations of resources.

Many of the services that the environment provides involve some kind of market failure, and hence the levels of provision in a market system will not be those corresponding to allocative efficiency. Much of resource and environmental economics is about devising ways to intervene in the market system so as to promote efficiency in the use of environmental services. In the next Part of the book we look at the problem of pollution, building on our preliminary discussion of that problem in this chapter under the externality rubric.

Further reading

For a thorough general coverage of welfare economics principles, see Bator (1957), Baumol (1977), Just *et al.* (1982), Kreps (1990), Varian (1987) or Layard and Walters (1978, chapter 1). Cornes and Sandler (1996) is an excellent advanced treatment of the welfare economics of public goods and externalities. Baumol and Oates (1988) develops the theory of environmental economics, with special attention to policy, from the welfare economics of public goods and externalities; see also Dasgupta (1990), Fisher (1981), Johansson (1987), Mäler (1985), and McInerney (1976). Verhoef (1999) is a recent survey of externality theory in relation to environmental economics, and Proost (1999) surveys contributions from public-sector economics. Classic early articles on environmental externalities include Ayres and Kneese (1969) and D'Arge and Kogiku (1972).

The analysis of democratic governance in terms of self-interested behaviour by politicians, voters, bureaucrats and pressure groups was systematically developed by Buchanan: see, for example, Buchanan and Tullock (1980). Renner (1999) derives some implications for sustainability policy from the work of the 'Virginia school' associated with Buchanan. Everret in Dietz *et al.* (1993) considers the history of environmental legislation in the USA in the period 1970 to 1990 within this framework.

Discussion questions

1. 'If the market puts a lower value on trees as preserved resources than as sources of timber for construction, then those trees should be felled for timber.' Discuss.

2. Do you think that individuals typically have enough information for it to make sense to have their preferences determine environmental policy?

3. How is the level of provision of national defence services, a public good, actually determined? Suggest a practical method for determining the level of provision that would satisfy an economist.

4. Economists see pollution problems as examples of the class of adverse externality phenomena. An adverse externality is said to occur when the decisions of one agent harm another in an unintended way, and when no compensation occurs. Does this mean that if a pollution source, such as a power station, compensates those affected by its emissions, then there is no pollution problem?

5. While some economists argue for the creation of private property rights to protect the environment, many of those concerned for the environment find this approach abhorrent. What are the essential issues in this dispute?

Problems

1. Suppose that a wood pulp mill is situated on a bank of the River Tay. The private marginal cost (MC) of producing wood pulp (in £ per ton) is given by the function

   $$\text{MC} = 10 + 0.5Y$$

   where Y is tons of wood pulp produced. In addition to this private marginal cost, an external cost is incurred. Each ton of wood pulp produces pollutant flows into the river which cause damage valued at £10. This is an external cost, as it is borne by the wider community but not by the polluting firm itself. The marginal benefit (MB) to society of each ton of produced pulp, in £, is given by

   $$\text{MB} = 30 - 0.5Y$$

a. Draw a diagram illustrating the marginal cost (MC), marginal benefit (MB), external marginal cost (EMC) and social marginal cost (SMC) functions.
b. Find the profit-maximising output of wood pulp, assuming the seller can obtain marginal revenue equal to the marginal benefit to society derived from wood pulp.
c. Find the pulp output which maximises social net benefits.
d. Explain why the socially efficient output of wood pulp is lower than the private profit-maximising output level.
e. How large would marginal external cost have to be in order for it to be socially desirable that no wood pulp is produced?

2. Demonstrate that equations 5.1 and 5.2 embody an assumption that there are no externalities in either consumption or production. Suppose that B's consumption of Y had a positive effect upon A's utility, and that the use of K by firm X adversely affects the output of firm Y. Show how the utility and production functions would need to be amended to take account of these effects.

3. In the chapter and in Appendix 5.3 we consider the two-person consumption-to-consumption externality. As invited in the Appendix, show that an efficient outcome could be realised if a planner required the sufferer to bribe the generator at the appropriate rate, and work out what that rate is.

4. In considering producer-to-consumer externalities in Appendix 5.3, it is stated that where there are multiple sources of emissions, and where only individuals suffer from pollution, each source should be taxed at the same rate. Prove this, and derive the tax rate.

5. Repeat Problem 4 for the case where pollution affects both lines of production as well as both individuals' utility.

Appendix 5.1 Conditions for efficiency and optimality

A5.1.1 Marginal rates of substitution and transformation

For an individual consumer the marginal rate of utility substitution, MRUS, between two commodities is defined as the rate at which one commodity can be substituted for the other, holding utility constant. For marginal changes in consumption levels, for $U = U(X, Y)$

$$\mathrm{d}U = U_X \mathrm{d}X + U_Y \mathrm{d}Y$$

where dU, dX and dY are differentials, and we are using U_X for $\partial U/\partial X$ and U_Y for $\partial U/\partial Y$, the marginal utilities. Setting dU = 0,

$$0 = U_X \mathrm{d}X + U_Y \mathrm{d}Y$$

so that

$$-U_Y \mathrm{d}Y = U_X \mathrm{d}X$$

and

$$-\mathrm{d}Y/\mathrm{d}X = U_X/U_Y$$

gives the MRUS as the ratio of the marginal utilities:

$$\mathrm{MRUS} = U_X/U_Y \tag{5.20}$$

The MRUS is the slope of the indifference curve at the relevant (X, Y) combination times −1. Since the slope is negative, the MRUS itself is positive, as it must be here given positive marginal utilities.

The marginal rate of technical substitution, MRTS, between two inputs to production is the rate at which one can be substituted for the other holding output constant. For marginal changes in input levels, for $X = X(K, L)$

$$\mathrm{d}X = X_K \mathrm{d}K + X_L \mathrm{d}L$$

where dX, dK and dL are differentials, and where $X_K = \partial X/\partial K$ and $X_L = \partial X/\partial L$ are the marginal products of capital and labour. Setting

$$\mathrm{d}X = 0$$

$$0 = X_K \mathrm{d}K + X_L \mathrm{d}L$$

$$-X_K \mathrm{d}K = X_L \mathrm{d}L$$

and

$$-\mathrm{d}K/\mathrm{d}L = X_L/X_K$$

gives the MRTS as the ratio of the marginal products of the labour and capital inputs:

$$\text{MRTS} = X_L/X_K \tag{5.21}$$

The MRTS is the slope of the isoquant at the relevant (K, L) combination times -1. Since the slope is negative, the MRTS itself is positive, as it must be here given positive marginal products.

The marginal rate of transformation, MRT, refers to the rate at which one commodity can be transformed into the other by means of marginal reallocations of one of the inputs to production. Thus MRT_K refers the effect on the output of Y when capital is, at the margin, shifted from use in the production of X to the production of Y, and MRT_L refers the effect on the output of Y when labour is, at the margin, shifted from use in the production of X to the production of Y. Consider shifting capital at the margin. For $X = X(K^X, L^X)$ and $Y = Y(K^Y, L^Y)$

$$\mathrm{d}X = X_K \mathrm{d}K^X + X_L \mathrm{d}L^X \text{ and } \mathrm{d}Y = Y_K \mathrm{d}K^Y + Y_L \mathrm{d}L^Y$$

where $\mathrm{d}K^X$, for example, is a marginal increase/decrease in the use of capital in the production of X. The definition of the marginal rate of transformation for capital is

$$\text{MRT}_K \equiv -\mathrm{d}Y/\mathrm{d}X$$

when there is no reallocation of labour. Note the use of the three-bar identity sign here to indicate a matter of definition. Then

$$\text{MRT}_K = -\left[\frac{Y_K \mathrm{d}K^Y + Y_L \mathrm{d}L^Y}{X_K \mathrm{d}K^X + X_L \mathrm{d}L^X}\right]$$

which for $\mathrm{d}L^Y = \mathrm{d}L^X = 0$ is

$$\text{MRT}_K = -\left[\frac{Y_K \mathrm{d}K^Y}{X_K \mathrm{d}K^X}\right]$$

and $\mathrm{d}K^Y = -\mathrm{d}K^X$, so

$$\text{MRT}_K = -\left[\frac{Y_K(-\mathrm{d}K^X)}{X_K \mathrm{d}K^X}\right]$$

where the $\mathrm{d}K^X$'s cancel, and taking account of the two minus signs we have

$$\text{MRT}_K = Y_K/X_K \tag{5.22a}$$

so that the marginal rate of transformation for capital is the ratio of the marginal products of capital in each line of production. A similar derivation, for $\mathrm{d}K^Y = \mathrm{d}K^X = 0$ and $\mathrm{d}L^Y = -\mathrm{d}L^X$, establishes that

$$\text{MRT}_L = Y_L/X_L \tag{5.22b}$$

A5.1.2 Efficiency conditions

Allocative efficiency exists when it is impossible to make one individual better off without making some other individual(s) worse off. We consider an economy with two individuals each consuming two commodities, where each commodity is produced by an industry comprising two firms, each of which uses two inputs – capital and labour.[12] For such an economy, the conditions characterising allocative efficiency can be derived by considering the following constrained maximisation problem:

$$\text{Max } U^{\mathrm{A}}(X^{\mathrm{A}}, Y^{\mathrm{A}})$$

subject to

$$U^{\mathrm{B}}(X^{\mathrm{B}}, Y^{\mathrm{B}}) = Z$$

$$X_1(K_1^X, L_1^X) + X_2(K_2^X, L_2^X) = X^{\mathrm{A}} + X^{\mathrm{B}}$$

$$Y_1(K_1^Y, L_1^Y) + Y_2(K_2^Y, L_2^Y) = Y^{\mathrm{A}} + Y^{\mathrm{B}}$$

$$K^{\mathrm{T}} = K_1^X + K_2^X + K_1^Y + K_2^Y$$

$$L^{\mathrm{T}} = L_1^X + L_2^X + L_1^Y + L_2^Y$$

We are looking for the conditions under which A's utility will be maximised, given that B's is held at some arbitrary level Z. The other constraints are that the total consumption of each commodity is equal to the amount produced, and that the sum of the capital and labour inputs across all firms is equal to the economy's respective endowments, K^{T} and L^{T}.

[12] Using two individuals, two commodities and two firms in each industry does not really involve any loss of generality. Exactly the same qualitative conditions in terms of marginal rates of substitution and transformation would emerge if we used *h* individuals, *n* commodities and *m* firms in each industry. Our analysis could be generalised by having individual utility depend also on labour supplied, so that the total amount of labour available to the economy would be a variable rather than a constraint. This would introduce additional conditions, but would not alter those derived here. Another direction of generalisation would be over time so that the availability of capital is a matter of choice rather than a constraint – Chapter 11 looks at intertemporal efficiency and optimality.

This problem can be dealt with using the Lagrangian method reviewed in Appendix 3.1. Here the Lagrangian is

$$
\begin{aligned}
L = {} & U^{A}(X^{A}, Y^{A}) + \lambda_1[U^{B}(X^{B}, Y^{B}) - Z] \\
& + \lambda_2[X_1(K_1^X, L_1^X) + X_2(K_2^X, L_2^X) - X^A - X^B] \\
& + \lambda_3[Y_1(K_1^Y, L_1^Y) + Y_2(K_2^Y, L_2^Y) - Y^A - Y^B] \\
& + \lambda_4[K^T - K_1^X - K_2^X - K_1^Y - K_2^Y] \\
& + \lambda_5[L^T - L_1^X - L_2^X - L_1^Y - L_2^Y]
\end{aligned}
$$

We now need a way of indicating the marginal product of an input to the production of a commodity in a particular firm. A straightforward extension of the notation already introduced here is to use, for example, X_K^1 for $\partial X_1/\partial K_1^X$, the marginal product of capital in the production of commodity X in firm 1 in the industry producing X.

In this notation, the first-order conditions are:

$$\frac{\partial L}{\partial X^A} = U_X^A - \lambda_2 = 0 \tag{5.23a}$$

$$\frac{\partial L}{\partial Y^A} = U_Y^A - \lambda_3 = 0 \tag{5.23b}$$

$$\frac{\partial L}{\partial X^B} = \lambda_1 U_X^B - \lambda_2 = 0 \tag{5.23c}$$

$$\frac{\partial L}{\partial Y^B} = \lambda_1 U_Y^B - \lambda_3 = 0 \tag{5.23d}$$

$$\frac{\partial L}{\partial K_1^X} = \lambda_2 X_K^1 - \lambda_4 = 0 \tag{5.23e}$$

$$\frac{\partial L}{\partial K_2^X} = \lambda_2 X_K^2 - \lambda_4 = 0 \tag{5.23f}$$

$$\frac{\partial L}{\partial L_1^X} = \lambda_2 X_L^1 - \lambda_5 = 0 \tag{5.23g}$$

$$\frac{\partial L}{\partial L_2^X} = \lambda_2 X_L^2 - \lambda_5 = 0 \tag{5.23h}$$

$$\frac{\partial L}{\partial K_1^Y} = \lambda_3 Y_K^1 - \lambda_4 = 0 \tag{5.23i}$$

$$\frac{\partial L}{\partial K_2^Y} = \lambda_3 Y_K^2 - \lambda_4 = 0 \tag{5.23j}$$

$$\frac{\partial L}{\partial L_1^Y} = \lambda_3 Y_L^1 - \lambda_5 = 0 \tag{5.23k}$$

$$\frac{\partial L}{\partial L_2^Y} = \lambda_3 Y_L^2 - \lambda_5 = 0 \tag{5.23l}$$

From equations a and b here

$$\frac{U_X^A}{U_Y^A} = \frac{\lambda_2}{\lambda_3} \tag{5.23m}$$

and from c and d

$$\frac{U_X^B}{U_Y^B} = \frac{\lambda_2/\lambda_1}{\lambda_3/\lambda_1} = \frac{\lambda_2}{\lambda_3} \tag{5.23n}$$

so that

$$\frac{U_X^A}{U_Y^A} = \frac{U_X^B}{U_Y^B}$$

which from equation 5.20 in Section A5.1.1 above is

$$\text{MRUS}^A = \text{MRUS}^B \tag{5.24}$$

which is the consumption efficiency condition stated as equation 5.3 in the text of the chapter.

Now, from equations 5.23e and 5.23f we have

$$X_K^1 = X_K^2 = \lambda_4/\lambda_2 \tag{5.23o}$$

from equations 5.23g and 5.23h

$$X_L^1 = X_L^2 = \lambda_5/\lambda_2 \tag{5.23p}$$

from equations 5.23i and 5.23j

$$Y_K^1 = Y_K^2 = \lambda_4/\lambda_3 \tag{5.23q}$$

and from equations 5.23k and 5.23l

$$Y_L^1 = Y_L^2 = \lambda_5/\lambda_3 \tag{5.23r}$$

From equations 5.23o and 5.23p

$$\frac{X_L^1}{X_K^1} = \frac{X_L^2}{X_K^2} = \frac{\lambda_5/\lambda_2}{\lambda_4/\lambda_2} = \frac{\lambda_5}{\lambda_4}$$

and from equations 5.23q and 5.23r

$$\frac{Y_L^1}{Y_K^1} = \frac{Y_L^2}{Y_K^2} = \frac{\lambda_5/\lambda_3}{\lambda_4/\lambda_3} = \frac{\lambda_5}{\lambda_4}$$

so that

$$\frac{X_L^1}{X_K^1} = \frac{X_L^2}{X_K^2} = \frac{Y_L^1}{Y_K^1} = \frac{Y_L^2}{Y_K^2} \tag{5.23s}$$

Recall from equation 5.21 in Section A5.1.1 above that for $X = X(K, L)$, MRTS $= X_L/X_K$. Hence, equation 5.23s here can be written as

$$\text{MRTS}_X^1 = \text{MRTS}_X^2 = \text{MRTS}_Y^1 = \text{MRTS}_Y^2 \quad (5.25)$$

where MRTS_X^1, for example, is the marginal rate of technical substitution for capital and labour in the production of commodity X by firm 1 in the X industry. What equation 5.25 says is (a) that all firms in an industry must have the same MRTS and (b) that the MRTS must be the same in all industries. The interpretation in the sense given by (b) means that equation 5.25 is equivalent to the production efficiency condition, equation 5.4, the intuition for which is found in the text of this chapter. It is (a) here that makes it legitimate to consider, as we did in the text, each industry as comprising a single firm.

Given that firms in the same industry operate with the same marginal products, we can write equations 5.23o to 5.23r as

$$X_K = \lambda_4/\lambda_2 \quad (5.23\text{t})$$

$$X_L = \lambda_5/\lambda_2 \quad (5.23\text{u})$$

$$Y_K = \lambda_4/\lambda_3 \quad (5.23\text{v})$$

and

$$Y_L = \lambda_5/\lambda_3 \quad (5.23\text{w})$$

Then, from equations 5.23v and 5.23t

$$\frac{Y_K}{X_K} = \frac{\lambda_4/\lambda_3}{\lambda_4/\lambda_2} = \frac{\lambda_2}{\lambda_3}$$

and from equations 5.23w and 5.23u

$$\frac{Y_L}{X_L} = \frac{\lambda_5/\lambda_3}{\lambda_5/\lambda_2} = \frac{\lambda_2}{\lambda_3}$$

so that

$$\frac{Y_K}{X_K} = \frac{Y_L}{X_L} = \frac{\lambda_2}{\lambda_3}$$

which from equations 5.22a and 5.22b in Section A5.1.1 above can be written as

$$\text{MRT}_L = \text{MRT}_K = \lambda_2/\lambda_3 \quad (5.23\text{x})$$

At equations 5.23m and 5.23n we obtained

$$\frac{U_X^A}{U_Y^A} = \frac{U_X^B}{U_Y^B} = \frac{\lambda_2}{\lambda_3}$$

which, by equation 5.20 from Section A5.1.1, is

$$\text{MRUS}^A = \text{MRUS}^B = \lambda_2/\lambda_3 \quad (5.23\text{y})$$

From equations 5.23x and 5.23y we get

$$\text{MRUS}^A = \text{MRUS}^B = \text{MRT}_L = \text{MRT}_K \quad (5.26)$$

which is the product-mix efficiency condition stated as equation 5.5 in the chapter.

A5.1.3 Optimality conditions

We now introduce a social welfare function, so as to derive the conditions that characterise an optimal allocation. Using the same assumptions about utility and production as in Section A5.1.2, the problem to be considered here is:

$$\text{Max } W\{U^A(X^A, Y^A), U^B(X^B, Y^B)\}$$

subject to

$$X_1(K_1^X, L_1^X) + X_2(K_2^X, L_2^X) = X^A + X^B$$

$$Y_1(K_1^Y, L_1^Y) + Y_2(K_2^Y, L_2^Y) = Y^A + Y^B$$

$$K^T = K_1^X + K_2^X + K_1^Y + K_2^Y$$

$$L^T = L_1^X + L_2^X + L_1^Y + L_2^Y$$

Here the Lagrangian is

$$\begin{aligned} L = {} & W\{U^A(X^A, Y^A), U^B(X^B, Y^B)\} \\ & + \lambda_2[X_1(K_1^X, L_1^X) + X_2(K_2^X, L_2^X) - X^A - X^B] \\ & + \lambda_3[Y_1(K_1^Y, L_1^Y) + Y_2(K_2^Y, L_2^Y) - Y^A - Y^B] \\ & + \lambda_4[K^T - K_1^X - K_2^X - K_1^Y - K_2^Y] \\ & + \lambda_5[L^T - L_1^X - L_2^X - L_1^Y - L_2^Y] \end{aligned}$$

where we have started numbering the multipliers at 2 so as to bring out more transparently the correspondences between the necessary conditions for efficiency and optimality – the fact that we use the same symbols and numbers in both cases does not, of course, mean that the multipliers take the same values in both cases.

The first-order conditions for this welfare maximisation problem are:

$$\frac{\partial L}{\partial X^A} = W_A U_X^A - \lambda_2 = 0 \quad (5.27\text{a})$$

$$\frac{\partial L}{\partial Y^A} = W_A U_Y^A - \lambda_3 = 0 \quad (5.27\text{b})$$

$$\frac{\partial L}{\partial X^B} = W_B U_X^B - \lambda_2 = 0 \quad (5.27\text{c})$$

$$\frac{\partial L}{\partial Y^{B}} = W_{B}U_{Y}^{B} - \lambda_3 = 0 \quad (5.27d)$$

$$\frac{\partial L}{\partial K_1^X} = \lambda_2 X_K^1 - \lambda_4 = 0 \quad (5.27e)$$

$$\frac{\partial L}{\partial K_2^X} = \lambda_2 X_K^2 - \lambda_4 = 0 \quad (5.27f)$$

$$\frac{\partial L}{\partial L_1^X} = \lambda_2 X_L^1 - \lambda_5 = 0 \quad (5.27g)$$

$$\frac{\partial L}{\partial L_2^X} = \lambda_2 X_L^2 - \lambda_5 = 0 \quad (5.27h)$$

$$\frac{\partial L}{\partial K_1^Y} = \lambda_3 Y_K^1 - \lambda_4 = 0 \quad (5.27i)$$

$$\frac{\partial L}{\partial K_2^Y} = \lambda_3 Y_K^2 - \lambda_4 = 0 \quad (5.27j)$$

$$\frac{\partial L}{\partial L_1^Y} = \lambda_3 Y_L^1 - \lambda_5 = 0 \quad (5.27k)$$

$$\frac{\partial L}{\partial L_2^Y} = \lambda_3 Y_L^2 - \lambda_5 = 0 \quad (5.27l)$$

where $W_A = \partial W/\partial U^A$ and $W_B = \partial W/\partial U^B$.

Note that equations e through to l in the set 5.27 are the same as e through to l in the set 5.23. It follows that optimality requires the efficiency in production condition, equation 5.25, rewritten here as

$$\text{MRTS}_X^1 = \text{MRTS}_X^2 = \text{MRTS}_Y^1 = \text{MRTS}_Y^2 \quad (5.28)$$

From a and b in set 5.27

$$\frac{U_X^A}{U_Y^A} = \frac{\lambda_2}{\lambda_3}$$

as W_A cancels. Similarly, from c and d in set 5.27,

$$\frac{U_X^B}{U_Y^B} = \frac{\lambda_2}{\lambda_3}$$

so that optimality requires

$$\frac{U_X^A}{U_Y^A} = \frac{U_X^B}{U_Y^B}$$

or

$$\text{MRUS}^A = \text{MRUS}^B = \lambda_2/\lambda_3 \quad (5.29)$$

which is the same as the consumption efficiency condition, 5.24, in the previous section.

From equations 5.27e through to 5.27l we can, as in the previous section, derive

$$\text{MRT}_L = \text{MRT}_K = \lambda_2/\lambda_3 \quad (5.30)$$

and from 5.29 and 5.30 we have

$$\text{MRUS}^A = \text{MRUS}^B = \text{MRT}_L = \text{MRT}_K \quad (5.31)$$

which is the same product mix condition as is required for efficiency.

Optimality requires the fulfilment of all of the efficiency conditions. In deriving the efficiency conditions, the utility of B is set at some arbitrary level. The maximisation problem considered there, as well as producing the conditions that any efficient allocation must satisfy, identifies the maximum level for A's utility conditional on the selected level of B's utility. In the welfare maximisation problem the function $W\{U^A, U^B\}$ selects the utility levels for A and B. As discussed in the text, only combinations of U^A and U^B that lie along the utility possibility frontier are relevant for welfare maximisation. All such combinations satisfy the efficiency conditions, and hence welfare maximisation entails satisfying the efficiency conditions as shown above. It also entails the condition stated as equation 5.7 in the chapter, which condition fixes the utility levels for A and B using the social welfare function.

From equations 5.27a through to 5.27d we have

$$W_A = \frac{\lambda_2}{U_X^A} \quad (5.32a)$$

$$W_A = \frac{\lambda_3}{U_Y^A} \quad (5.32b)$$

$$W_B = \frac{\lambda_2}{U_X^B} \quad (5.32c)$$

$$W_B = \frac{\lambda_3}{U_Y^B} \quad (5.32d)$$

From a and c here we get

$$\frac{W_A}{W_B} = \frac{U_X^B}{U_X^A}$$

and from b and d we get

$$\frac{W_A}{W_B} = \frac{U_Y^B}{U_Y^A}$$

so that

$$\frac{W_A}{W_B} = \frac{U_X^B}{U_X^A} = \frac{U_Y^B}{U_Y^A} \quad (5.33)$$

which is equation 5.7 in the chapter.

The SWF is $W = W(U^A, U^B)$ so that

$$dW = W_A dU^A + W_B dU^B$$

Setting the left-hand side here equal to zero so as to consider small movements along a social welfare indifference curve, and rearranging, gives

$$-\frac{dU^B}{dU^A} = \frac{W_A}{W_B}$$

for the slope of a social welfare indifference curve. The slope of the utility possibility frontier is $-dU^B/dU^A$ which is equal to U_X^B/U_X^A and to U_Y^B/U_Y^A.

Appendix 5.2 Market outcomes

In this appendix we establish that, given the 'ideal' institutional conditions set out in the text of the chapter, a system of markets will bring about the satisfaction of the necessary conditions for efficiency in allocation – the consumption efficiency condition, the production efficiency condition and the product-mix condition.

A5.2.1 Individuals: utility maximisation

Consider an individual consumer, with a fixed money income M and gaining utility from the consumption of two goods, X and Y. The prices of these goods are determined in competitive markets, at the levels P_X and P_Y, and are taken as given by all individuals. With this individual's utility function given by

$$U = U(X, Y)$$

we can express the problem of maximising utility subject to a budget constraint as

$$\text{Max } U(X, Y)$$

subject to

$$P_X X + P_Y Y = M$$

The Lagrangian for this problem is

$$L = U(X, Y) + \lambda[P_X X + P_Y Y - M]$$

and, using the same notation for the derivatives (the marginal utilities) as previously, the first-order conditions for a maximum are:

$$\frac{\partial L}{\partial X} = U_X + \lambda P_X = 0 \quad (5.34a)$$

$$\frac{\partial L}{\partial X} = U_Y + \lambda P_Y = 0 \quad (5.34b)$$

From these equations we get

$$U_X = -\lambda P_X$$

$$U_Y = -\lambda P_Y$$

so that

$$\frac{U_X}{U_Y} = \frac{P_X}{P_Y} \quad (5.35)$$

Equation 5.35 holds for all consumers, all of whom face the same P_X and P_Y, and the left-hand side is the marginal rate of utility substitution. So, for any two consumers A and B, we have:

$$\text{MRUS}^A = \text{MRUS}^B = \frac{P_X}{P_Y} \quad (5.36)$$

The consumption efficiency condition is satisfied, see equation 5.3 in the chapter and equation 5.24 in the previous appendix, and the marginal rate of utility substitution common to all individuals is equal to the price ratio, as stated in the chapter at equation 5.8.

A5.2.2 Firms: profit maximisation

Consider the production of X by firms $i = 1, 2, \ldots, m$. All firms face the same selling price, P_X, and all pay the same fixed prices for capital and labour inputs, P_K and P_L. The objective of every firm is to maximise profit, so to ascertain the conditions characterising the behaviour of the ith firm we consider

$$\text{Max} \quad P_X X_i(K_i^X, L_i^X) - P_K K_i^X - P_L L_i^X$$

where the necessary conditions are

$$P_X X_K^i - P_K = 0 \tag{5.37a}$$

$$P_X X_L^i - P_L = 0 \tag{5.37b}$$

or

$$X_K^i = \frac{P_K}{P_X} \tag{5.38a}$$

$$X_L^i = \frac{P_L}{P_X} \tag{5.38b}$$

from which

$$\frac{X_K^i}{X_L^i} = \frac{P_K}{P_L} \tag{5.39}$$

Equation 5.39 holds for all i, and the left-hand side is the expression for the marginal rate of technical substitution. Hence, all firms producing X operate with the same MRTS. Further, it is obvious that considering profit maximisation by the jth firm in the industry producing the commodity Y will lead to

$$\frac{Y_K^j}{Y_L^j} = \frac{P_K}{P_L} \tag{5.40}$$

which with equation 5.39 implies

$$\text{MRTS}_X^i = \text{MRTS}_Y^j \tag{5.41}$$

for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The production efficiency condition, equation 5.4 in the chapter, is satisfied.

Recall that

$$\text{MRT}_K = \frac{Y_K}{X_K} \tag{5.42a}$$

and

$$\text{MRT}_L = \frac{Y_L}{X_L} \tag{5.42b}$$

From equations 5.38a and 5.38b, and the corresponding conditions from profit maximisation in the production of Y, omitting the superscripts for firms we have

$$X_K = \frac{P_K}{P_X},\ X_L = \frac{P_L}{P_X},\ Y_K = \frac{P_K}{P_Y},\ Y_L = \frac{P_L}{P_Y}$$

and substituting and cancelling in equations 5.42a and 5.42b,

$$\text{MRT}_K = \frac{P_X}{P_Y} = \text{MRT}_L$$

and bringing this together with equation 5.36 gives

$$\text{MRUS}^{\text{A}} = \text{MRUS}^{\text{B}} = \text{MRT}_K = \text{MRT}_L \tag{5.43}$$

which shows that the product-mix condition, equation 5.5 in the chapter and 5.26 in the previous appendix, is satisfied.

In the chapter it was stated that the necessary condition for profit maximisation was the equality of marginal cost with the output price. To establish this let $C(X^i)$ be the firm's cost function and write the profits for the ith firm in the industry producing X as

$$\pi_X^i = P_X X^i - C(X^i)$$

from which the necessary condition for maximisation is

$$\partial \pi_X^i / \partial X^i = P_X - \partial C / \partial X^i = 0$$

which is

$$P_X = \partial C / \partial X^i$$

i.e. price equals marginal cost.

Appendix 5.3 Market failure

A5.3.1 Public goods

In the two-person, two-commodity, two-resource economy considered in the preceding appendix, now let X be a public good and Y a private good. Given the results established there regarding the conditions for efficiency in relation to firms in the same industry, we can simplify here without loss by assuming that each commodity is produced in an industry which has just one firm. Given that we are taking the defining characteristic of a public good to be that it is consumed in the same quantity by all, we can state the problem from which the necessary conditions for efficiency are to be derived as:

$$\text{Max } U^{A}(X, Y^{A})$$

subject to

$$U^{B}(X, Y^{B}) = Z$$

$$X(K^{X}, L^{X}) = X$$

$$Y(K^{Y}, L^{Y}) = Y^{A} + Y^{B}$$

$$K^{T} = K^{X} + K^{Y}$$

$$L^{T} = L^{X} + L^{Y}$$

The Lagrangian for this problem is

$$\begin{aligned} L = U^{A}(X, Y^{A}) &+ \lambda_1[U^{B}(X, Y^{B}) - Z] \\ &+ \lambda_2[X(K^{X}, L^{X}) - X] \\ &+ \lambda_3[Y(K^{Y}, L^{Y}) - Y^{A} - Y^{B}] \\ &+ \lambda_4[K^{T} - K^{X} - K^{Y}] \\ &+ \lambda_5[L^{T} - L^{X} - L^{Y}] \end{aligned}$$

from which the necessary conditions for maximisation are:

$$\frac{\partial L}{\partial X} = U_X^A + \lambda_1 U_X^B - \lambda_2 = 0 \tag{5.44a}$$

$$\frac{\partial L}{\partial Y^A} = U_Y^A - \lambda_3 = 0 \tag{5.44b}$$

$$\frac{\partial L}{\partial Y^B} = \lambda_1 U_Y^B - \lambda_3 = 0 \tag{5.44c}$$

$$\frac{\partial L}{\partial K^X} = \lambda_2 X_K - \lambda_4 = 0 \tag{5.44d}$$

$$\frac{\partial L}{\partial L^X} = \lambda_2 X_L - \lambda_5 = 0 \tag{5.44e}$$

$$\frac{\partial L}{\partial K^Y} = \lambda_3 Y_K - \lambda_4 = 0 \tag{5.44f}$$

$$\frac{\partial L}{\partial L^Y} = \lambda_3 Y_L - \lambda_5 = 0 \tag{5.44g}$$

Consider first equations 5.44d to 5.44g, which relate to production. They imply

$$\frac{X_L}{X_K} = \frac{\lambda_5}{\lambda_4} = \frac{Y_L}{Y_K}$$

which is

$$\text{MRTS}_X = \text{MRTS}_Y$$

so that production efficiency is required. They also imply

$$\frac{Y_K}{X_K} = \frac{\lambda_2}{\lambda_3} = \frac{Y_L}{X_L}$$

which is

$$\text{MRT}_K = \text{MRT}_L = \frac{\lambda_2}{\lambda_3} \tag{5.45}$$

so that as regards production activities, the conditions in the presence of a public good are the same as in the standard case, see Appendix 5.1, where there are no public goods.

Now consider equations 5.44a to 5.44c, which relate to consumption. From equations a and b there

$$\frac{U_X^A}{U_Y^A} = \frac{\lambda_2}{\lambda_3} - \frac{\lambda_1 U_X^B}{\lambda_3} \tag{5.46a}$$

and using equation 5.44c we can write

$$\frac{U_X^B}{U_Y^B} = \frac{U_X^B}{\lambda_3/\lambda_1} = \frac{\lambda_1 U_X^B}{\lambda_3} \tag{5.46b}$$

and adding 5.46a and 5.46b gives:

$$\frac{U_X^A}{U_Y^A} + \frac{U_X^B}{U_Y^B} = \frac{\lambda_2}{\lambda_3} - \frac{\lambda_1 U_X^B}{\lambda_3} + \frac{\lambda_1 U_X^B}{\lambda_3} = \frac{\lambda_2}{\lambda_3} \tag{5.47}$$

Using the definition for MRUS, equation 5.47 is

$$\text{MRUS}^A + \text{MRUS}^B = \frac{\lambda_2}{\lambda_3}$$

so that from equation 5.45 we have the condition

$$\text{MRUS}^A + \text{MRUS}^B = \text{MRT} \tag{5.48}$$

stated as equation 5.15 in the chapter.

A5.3.2 Externalities: consumer to consumer

As in the text, we ignore production in looking at this case. Given that we have not previously looked at a pure exchange economy, it will be convenient first to look at such an economy where there is no external effect.

To identify the necessary conditions for efficiency, we look at

$$\text{Max } U^{A}(X^{A}, Y^{A})$$

subject to

$$U^B(X^B, Y^B) = Z$$

$$X^T = X^A + X^B$$

$$Y^T = Y^A + Y^B$$

where X^T and Y^T are the total amounts of the two commodities to be allocated as between A and B. The Lagrangian for this problem is

$$\begin{aligned} L = U^A(X, Y^A) + \lambda_1[U^B(X, Y^B) - Z] \\ + \lambda_2[X^T - X^A - X^B] \\ + \lambda_3[Y^T - Y^A - Y^B] \end{aligned}$$

and the necessary conditions are

$$\frac{\partial L}{\partial X^A} = U_X^A - \lambda_2 = 0$$

$$\frac{\partial L}{\partial Y^A} = U_Y^A - \lambda_3 = 0$$

$$\frac{\partial L}{\partial X^B} = \lambda_1 U_X^B - \lambda_2 = 0$$

$$\frac{\partial L}{\partial Y^B} = \lambda_1 U_Y^B - \lambda_3 = 0$$

from which we get

$$\frac{U_X^A}{U_Y^A} = \frac{U_X^B}{U_Y^B} = \frac{\lambda_2}{\lambda_3}$$

which is the same consumption efficiency condition as for the economy with production, i.e. $\text{MRUS}^A = \text{MRUS}^B$. We already know, from Appendix 5.2, that consumers facing given and fixed prices P_X and P_Y and maximising utility subject to a budget constraint will satisfy this condition.

Now, suppose that B's consumption of Y is an argument in A's utility function. We are assuming that Y^B is a source of disutility to A. Then the maximisation problem to be considered is

$$\text{Max } U^A(X^A, Y^A, Y^B)$$

subject to

$$U^B(X^B, Y^B) = Z$$

$$X^T = X^A + X^B$$

$$Y^T = Y^A + Y^B$$

for which the Lagrangian is

$$\begin{aligned} L = U^A(X^A, Y^A, Y^B) + \lambda_1[U^B(X^B, Y^B) - Z] \\ + \lambda_2[X^T - X^A - X^B] \\ + \lambda_3[Y^T - Y^A - Y^B] \end{aligned}$$

with necessary conditions

$$\frac{\partial L}{\partial X^A} = U_X^A - \lambda_2 = 0 \tag{5.49a}$$

$$\frac{\partial L}{\partial Y^A} = U_Y^A - \lambda_3 = 0 \tag{5.49b}$$

$$\frac{\partial L}{\partial X^B} = \lambda_1 U_X^B - \lambda_2 = 0 \tag{5.49c}$$

$$\frac{\partial L}{\partial Y^B} = U_{YB}^A \lambda_1 U_Y^B - \lambda_3 = 0 \tag{5.49d}$$

where $U_{YB}^A = \partial U^A/\partial Y^B$. Note that Y^B is a source of disutility to A so that $U_{YB}^A < 0$. From 5.49a and 5.49b we get

$$\frac{U_X^A}{U_Y^A} = \frac{\lambda_2}{\lambda_3} \tag{5.50a}$$

from 5.49c

$$U_X^B = \frac{\lambda_2}{\lambda_1} \tag{5.50b}$$

and from 5.49d

$$U_Y^B = \frac{\lambda_3}{\lambda_1} - \frac{U_{YB}^A}{\lambda_1} \tag{5.50c}$$

so that, using 5.50b and 5.50c,

$$\frac{U_X^B}{U_Y^B} = \frac{\lambda_3}{\lambda_3 - U_{BY}^A} \tag{5.50d}$$

Looking at 5.50a and 5.50d we see that with the externality, efficiency does not require the condition $\text{MRUS}^A = \text{MRUS}^B$. But we have just seen that, facing just the prices P_X and P_Y, market trading between A and B will give $\text{MRUS}^A = \text{MRUS}^B$. So, given the existence of this externality, market exchange will not satisfy the conditions, 5.50a and 5.50d, for efficiency.

Suppose now that there exists a central planner who knows the two agents' utility functions and the quantities of X and Y available. The planner's objective is an efficient allocation, to be realised by the two agents individually maximising utility on terms set by the planner, rather by the planner telling the agents at what levels to consume. The planner declares prices P_X and P_Y, and also requires B to compensate A for her Y^B suffering at the rate c per unit of Y^B. In that case, A's utility maximisation problem is

$$\text{Max } U^A(X^A, Y^A, Y^B)$$

subject to

$$P_X X^A + P_Y Y^A = M^A + cY^B$$

where M^A is A's income before the receipt of any compensation from B. The Lagrangian for this problem is:

$$L = U^A(X^A, Y^A, Y^B) + \lambda_A[P_X X^A + P_Y Y^A - M^A - cY^B]$$

Note that Y^B is not a choice variable for A. The level of Y^B is chosen by B. The necessary conditions for A's maximisation problem are

$$\frac{\partial L}{\partial X^A} = U_X^A + \lambda_A P_X = 0$$

$$\frac{\partial L}{\partial Y^A} = U_Y^A + \lambda_A P_Y = 0$$

from which

$$\frac{U_X^A}{U_Y^A} = \frac{P_X}{P_Y} \tag{5.51a}$$

B's utility maximisation problem is

$$\text{Max } U^B(X^B, Y^B)$$

subject to

$$P_X X^B + P_Y Y^B = M^B - cY^B$$

the Lagrangian for which is

$$L = U^B(X^B, Y^B) + \lambda_B[P_X X^B + P_Y Y^B - M^B + cY^B]$$

with necessary conditions

$$\frac{\partial L}{\partial X^B} = U_X^B + \lambda_B P_X = 0$$

$$\frac{\partial L}{\partial Y^B} = U_Y^B + \lambda_B P_Y + \lambda_B c = 0$$

from which

$$\frac{U_X^B}{U_Y^B} = \frac{P_X}{P_Y + c} \tag{5.51b}$$

So, we have 5.50a and 5.50d as the efficiency conditions and 5.51a and 5.51b as the individual utility-maximising conditions. Comparing 5.50a and 5.50d with 5.51a and 5.51b, it will be seen that they are the same for:

$$\lambda_2 = P_X,\ \lambda_3 = P_Y \text{ and } c = -U_{YB}^A$$

If, that is, the planner solves the appropriate maximisation problem and sets P_X and P_Y at the shadow prices of the commodities, and requires B to compensate A at a rate which is equal to, but of opposite sign to, A's marginal disutility in respect of the external effect, then A and B individually maximising utility given those prices and that compensation rate will bring about an efficient allocation. The planner is putting a price on the external effect, and the required price is A's marginal disutility.

However, as shown in the discussion of the Coase theorem in the body of the chapter, it is not actually necessary to have this kind of intervention by the planner. If A had the legal right to extract full compensation from B, had a property right in an unpolluted environment, then the right price for efficiency would emerge as the result of bargaining between A and B.

In considering the consumption-to-consumption case in the chapter we argued that the liability/property right could be assigned the other way round and still bring about an efficient outcome. The corresponding procedure with a planner setting the terms on which the two agents maximised utility would be to have the planner work out what Y^B would be with the externality uncorrected, say Y^{B*}, and then require A to compensate B for reducing Y^B below that level. In that case, A's maximisation problem would be

$$\text{Max } U^A(X^A, Y^A, Y^B)$$

subject to

$$P_X X^A + P_Y Y_A = M^A - b(Y^{B*} - Y^B)$$

and B's would be

$$\text{Max } U^B(X^B, Y^B)$$

subject to

$$P_X X^B + P_Y Y^B = M^B + b(Y^{B*} - Y^B)$$

where we use b for 'bribe'. It is left as an exercise to confirm that this arrangement would, given suitable P_X, P_Y and b, produce an efficient outcome.

The situation considered in the chapter actually differed from that considered here in a couple of respects. First, in that example the external effect involved A doing something – playing a musical instrument – which did not have a price attached to it, and which B did not do. In the uncorrected externality situation there, A pursued the 'polluting' activity up to the level where its marginal utility was zero. In the chapter, we considered things in terms of monetary costs and benefits in a partial equilibrium context, rather than utility maximisation in a general equilibrium context. Thinking about that noise pollution example in the following way may help to make the connections, and make a further point.

Let Y^A be the number of hours that A plays her instrument. Consider each individual's utility to depend on income and Y^A, so that $U^A = U^A(M^A, Y^A)$ and $U^B = U^B(M^B, Y^A)$, where $\partial U^A/\partial Y^A > 0$ and $\partial U^B/\partial Y^A < 0$. Consider welfare maximisation for given M^A and M^B. The problem is

$$\text{Max } W\{U^A(M^A, Y^A), U^B(M^B, Y^A)\}$$

where the only choice variable is Y^A, so that the necessary condition is:

$$W_A U^A_{YA} = -W_B U^B_{YA}$$

For equal welfare weights, this is

$$U^A_{YA} = -U^B_{YB}$$

or

Marginal benefit of music to A = Marginal cost of music to B

which is the condition as stated in the chapter. The further point that the derivation of this condition here makes is that the standard simple story about the Coase theorem implicitly assigns equal welfare weights to the two individuals.

A5.3.3 Externalities: producer to producer

To begin here, we suppose that the production function for Y is

$$Y = Y(K^Y, L^Y, S) \text{ with } Y_S = \partial Y/\partial S > 0$$

and for X is

$$X = X(K^X, L^X, S) \text{ with } X_S = \partial X/\partial S < 0$$

where S is pollutant emissions arising in the production of Y and adversely affecting the production of X. The Lagrangian from which the conditions for efficiency are to be derived is:

$$\begin{aligned} L = U^A(X^A, Y^A) &+ \lambda_1[U^B(X^B, Y^B) - Z] \\ &+ \lambda_2[X(K^X, L^X, S) - X^A - X^B] \\ &+ \lambda_3[Y(K^Y, L^Y, S) - Y^A - Y^B] \\ &+ \lambda_4[K^T - K^X - K^Y] \\ &+ \lambda_5[L^T - L^X - L^Y] \end{aligned}$$

The reader can readily check that in this case, taking derivates of L with respect to X^A, Y^A, X^B, Y^B, K^X, L^X, K^Y and L^Y gives, allowing for the fact that there is just one firm in each industry, the consumption, production and product-mix conditions derived in Section A5.1.2 and stated in the chapter. Taking the derivative of L with respect to S gives the additional condition

$$\frac{\partial L}{\partial S} = \lambda_2 X_S + \lambda_3 Y_S = 0$$

or

$$\frac{\lambda_2}{\lambda_3} = -\frac{Y_S}{X_S} \tag{5.52}$$

Now, suppose that a central planner declares prices $P_X = \lambda_2$, $P_Y = \lambda_3$, $P_K = \lambda_4$, $P_L = \lambda_5$, and requires that the firm producing Y pay compensation to the firm affected by its emissions at the rate c per unit S. Then, the Y firm's problem is

$$\text{Max } P_Y Y(K^Y, L^Y, S) - P_K K^Y - P_L L^Y - cS$$

with the usual necessary conditions

$$P_Y Y_K - P_K = 0$$

$$P_Y Y_L - P_L = 0$$

plus

$$P_Y Y_S - c = 0 \tag{5.53}$$

Compare equation 5.52 with 5.53. If we set $c = -P_X X_S$ then the latter becomes

$$P_Y Y_S = -P_X X_S$$

or

$$\frac{P_X}{P_Y} = -\frac{Y_S}{X_S} \tag{5.54}$$

which, for $P_X = \lambda_2$ and $P_Y = \lambda_3$, is the same as equation 5.52. With this compensation requirement in place, the profit-maximising behaviour of the Y firm will be as required for efficiency. Note that the rate of compensation makes sense. $P_X X_S$ is the reduction in X's profit for a given level of output when Y increases S. Note also that while we have called this charge on emissions of S by the Y firm 'compensation', we have not shown that efficiency requires that the X firm actually receives such compensation. The charge c, that is, might equally well be collected by the planner, in which case we would call it a tax on emissions.[13]

In the chapter we noted that one way of internalising a producer-to-producer externality could be for the firms to merge, or to enter into an agreement to maximise joint profits. A proof of this claim is as follows. The problem then is

$$\text{Max } P_X X(K^X, L^X, S) + P_Y(K^Y, L^Y, S) \\ - P_K(K^X + K^Y) - P_L(L^X + L^Y)$$

for which the necessary conditions are

$$P_X X_K - P_K = 0$$

$$P_X X_L - P_L = 0$$

$$P_Y Y_K - P_K = 0$$

$$P_Y Y_L - P_L = 0$$

which, given $P_X = \lambda_2$, $P_K = \lambda_4$ etc., satisfy the standard (no externality) efficiency conditions, plus

$$P_X X_S + P_Y Y_S = 0$$

This last condition for joint profit maximisation can be written as

$$\frac{P_X}{P_Y} = -\frac{Y_S}{X_S}$$

which is just equation 5.54, previously shown to be necessary, in addition to the standard conditions, for efficiency in the presence of this kind of externality.

In Chapter 2 we noted that the fact that matter can neither be created nor destroyed is sometimes overlooked in the specification of economic models. We have just been guilty in that way ourselves – writing

$$Y = Y(K^Y, L^Y, S)$$

with S as some kind of pollutant emission, has matter, S, appearing from nowhere, when, in fact, it must have a material origin in some input to the production process. A more satisfactory production function for the polluting firm would be

$$Y = Y(K^Y, L^Y, R^Y, S\{R^Y\})$$

where R^Y is the input of some material, say tonnes of coal, and $S\{R^Y\}$ maps coal burned into emissions, of say smoke, and $\partial Y/\partial R^Y = Y_R > 0$, $\partial Y/\partial S = Y_S > 0$ and $\partial S/\partial R^Y = S_{RY} > 0$. We shall now show that while this more plausible model specification complicates the story a little, it does not alter the essential message.

To maintain consistency with the producer-to-producer case as analysed above, and in the chapter, we will assume that in the production of X the use of R does not give rise to emissions of smoke. Then, the Lagrangian for deriving the efficiency conditions is:

$$\begin{aligned} L = {} & U^A(X^A, Y^A) + \lambda_1[U^B(X^B, Y^B) - Z] \\ & + \lambda_2[X(K^X, L^X, R^X, S\{R^Y\}) - X^A - X^B] \\ & + \lambda_3[Y(K^Y, L^Y, R^Y, S\{R^Y\}) - Y^A - Y^B] \\ & + \lambda_4[K^T - K^X - K^Y] \\ & + \lambda_5[L^T - L^X - L^Y] \\ & + \lambda_6[R^T - R^X - R^Y] \end{aligned}$$

In the production function for X, $\partial X/\partial R^X = X_R > 0$ and $\partial X/\partial S = X_S < 0$. The reader can confirm that taking derivatives here with respect to all the choice variables except R^X and R^Y gives all of the standard conditions. Then, with respect to R^X and R^Y, we get

[13] However, if c takes the form of a tax rather than compensation paid to the X firm, the question arises as to what happens to the tax revenue. It cannot remain with the planner, otherwise the government, as the planner does not count as an agent. If the planner/government has unspent revenues, it would be possible to make some agent better off without making any other agent(s) worse off. Given the simple model specification here, where, for example, there is no tax/welfare system and no public goods supply, we cannot explore this question further. It is considered, for example, in Chapter 4 of Baumol and Oates (1988), and the 'double dividend' literature reviewed in Chapter 10 below is also relevant.

$$\frac{\partial L}{\partial R^X} = \lambda_2 X_R - \lambda_6 = 0 \tag{5.55a}$$

$$\frac{\partial L}{\partial R^Y} = \lambda_2 X_S S_{RY} + \lambda_3 Y_R + \lambda_3 Y_S S_{RY} - \lambda_6 = 0 \tag{5.55b}$$

As before, suppose a planner sets $P_X = \lambda_2, \ldots, P_L = \lambda_5$ plus $P_R = \lambda_6$ and a tax on the use of R in the production of Y at the rate t. Then the profit maximisation problem for the firm producing Y is

$$\text{Max } P_Y Y(K^Y, L^Y, R^Y, S\{R^Y\}) - P_K K^Y - P_L L^Y - P_R R^Y - tR^Y$$

and for the firm producing X it is

$$\text{Max } P_X X(K^X, L^X, R^X, S\{R^Y\}) - P_K K^X - P_L L^X - P_R R^X$$

If the reader derives the necessary conditions here, which include

$$P_Y Y_R + P_Y Y_S S_{RY} - P_R - t = 0 \tag{5.56}$$

you can verify that for $P_X = \lambda_2, \ldots, P_L = \lambda_5$ and $P_R = \lambda_6$ with

$$t = -P_X X_S S_{RY} \tag{5.57}$$

independent profit maximisation by both firms satisfies the standard efficiency conditions plus the externality correction conditions stated above as equations 5.55a and 5.55b. The rationale for this rate of tax should also be apparent: S_{RY} is the increase in smoke for an increase in Y's use of R, X_S gives the effect of more smoke on the output of X for given K^X and L^X, and P_X is the price of X.

Now consider joint profit maximisation. From

$$\text{Max } P_X X(K^X, L^X, R^X, S\{R^Y\}) + P_Y(K^Y, L^Y, R^Y, S\{R^Y\}) - P_K(K^X + K^Y) - P_L(L^X + L^Y) - P_R(R^X + R^Y)$$

the necessary conditions are

$$P_X X_K - P_K = 0$$

$$P_X X_L - P_L = 0$$

$$P_Y Y_K - P_K = 0$$

$$P_Y Y_L - P_L = 0$$

$$P_X X_R - P_R = 0$$

$$P_Y Y_R + P_Y Y_S S_{RY} + P_X X_S S_{RY} - P_R = 0$$

Substituting from equation 5.57 into 5.56 for t gives the last of these equations, showing that the outcome under joint profit maximisation is the same as with the tax on the use of R in the production of Y.

A5.3.4 Externalities: producer to consumers

The main point to be made for this case concerns the implications of non-rivalry and non-excludability. These are not peculiar to the producer-to-consumers case, but are conveniently demonstrated using it. To simplify the notation, we revert to having emissions in production occur without any explicit representation of their material origin. As noted in the analysis of the producer-to-producer case, this simplifies without, for present purposes, missing anything essential. We assume that the production of Y involves pollutant emissions which affect both A and B equally, though, of course, A and B might have different preferences over pollution and commodities. Pollution is, that is, in the nature of a public bad – A/B's consumption is non-rival with respect to B/A's consumption, and neither can escape, be excluded from, consumption.

The Lagrangian for the derivation of the efficiency conditions is

$$\begin{aligned} L = U^A(X^A, Y^A, S) &+ \lambda_1[U^B(X^B, Y^B, S) - Z] \\ &+ \lambda_2[X(K^X, L^X) - X^A - X^B] \\ &+ \lambda_3[Y(K^Y, L^Y, S) - Y^A - Y^B] \\ &+ \lambda_4[K^T - K^X - K^Y] \\ &+ \lambda_5[L^T - L^X - L^Y] \end{aligned}$$

where $\partial U^A/\partial S = U_S^A < 0$, $\partial U^B/\partial S = U_S^B < 0$ and $\partial Y/\partial S = Y_S > 0$. The necessary conditions are:

$$\frac{\partial L}{\partial X^A} = U_X^A - \lambda_2 = 0 \tag{5.58a}$$

$$\frac{\partial L}{\partial Y^A} = U_Y^A - \lambda_3 = 0 \tag{5.58b}$$

$$\frac{\partial L}{\partial X^B} = \lambda_1 U_X^B - \lambda_2 = 0 \tag{5.58c}$$

$$\frac{\partial L}{\partial Y^B} = \lambda_1 U_Y^B - \lambda_3 = 0 \tag{5.58d}$$

$$\frac{\partial L}{\partial S} = U_S^A + \lambda_1 U_S^B + \lambda_3 Y_S = 0 \tag{5.58e}$$

$$\frac{\partial L}{\partial K^X} = \lambda_2 X_K - \lambda_4 = 0 \quad (5.58f)$$

$$\frac{\partial L}{\partial L^X} = \lambda_2 X_L - \lambda_5 = 0 \quad (5.58g)$$

$$\frac{\partial L}{\partial K^Y} = \lambda_3 Y_K - \lambda_4 = 0 \quad (5.58h)$$

$$\frac{\partial L}{\partial L^Y} = \lambda_3 Y_L - \lambda_5 = 0 \quad (5.58i)$$

The reader can check that these can be expressed as the standard consumption, production and product-mix conditions plus

$$U_S^A + \lambda_1 U_S^B = -\lambda_3 Y_S \quad (5.59)$$

from equation 5.58e.

Now suppose that a central planner declares prices $P_X = \lambda_2$, $P_Y = \lambda_3$, $P_K = \lambda_4$ and $P_Y = \lambda_5$. Proceeding as done previously in this appendix, the reader can check that utility and profit maximisation at these prices will satisfy all of the standard conditions, but not equation 5.59. Suppose then that the planner also requires the producer of Y to pay a tax at the rate t on emissions of S. Considering

$$\text{Max } P_Y Y(K^Y, L^Y, S) - P_K K^Y - P_L L^Y - tS$$

gives the standard conditions

$$P_Y Y_K - P_K = 0$$

$$P_L L_Y - P_L = 0$$

plus

$$P_Y Y_S - t = 0$$

which can be written as

$$t = \lambda_3 Y_S \quad (5.60)$$

Comparing equations 5.59 and 5.60, we have the result that, in this case, achieving efficiency as the result of individual utility and profit maximisation requires, in addition to the usual 'ideal' institutional arrangements, that the producer of Y faces an emissions tax at the rate:

$$t = -[U_S^A + \lambda_1 U_S^B] \quad (5.61)$$

Note that since U_S^A and U_S^B are both negative, the tax rate required is positive.

In the chapter, we stated that the correction of this kind of externality required that the tax rate be set equal to the marginal external cost at the efficient allocation. We will now show that this is exactly what the result 5.61 requires. From equation 5.58c

$$\lambda_1 = \frac{\lambda_2}{U_X^B}$$

and from equation 5.58a

$$1 = \frac{\lambda_2}{U_X^A}$$

so that equation 5.61 can be written

$$t = -\left[\frac{\lambda_2}{U_X^A} U_S^A + \frac{\lambda_2}{U_X^B} U_S^B\right]$$

which, using $P_X = \lambda_2$, is

$$t = -P_X\left[\frac{U_S^A}{U_X^A} + \frac{U_S^B}{U_X^B}\right]$$

or

$$t = P_X \lfloor \text{MRUS}_{XS}^A + \text{MRUS}_{XS}^B \rfloor \quad (5.62)$$

as stated at equation 5.17 in the chapter.[14] The tax rate is the monetary value of the increases in X consumption that would be required to hold each individual's utility constant in the face of a marginal increase in S. We could, of course, have derived the marginal external cost in terms of Y, rather than X, compensation.

In this case, the joint profit maximisation solution is clearly not, even in principle, available for the correction of the market failure problem. Nor, given the public good characteristic of the suffering of A and B, is the property rights/legal liability solution. The way to correct this kind of market failure is to tax the emissions at a rate which is equal to the marginal

[14] To recapitulate, the marginal rate of substitution here is derived as follows. For $U(X, Y, S)$

$$dU = U_X dX + U_Y dY + U_S dS$$

so for dU and $dY = 0$

$$0 = U_X dX + U_S dS$$

and

$$\frac{U_S}{U_X} = -\frac{dX}{dS}$$

external cost arising at the efficient allocation. It can be shown that where there is more than one source of the emissions, all sources are to be taxed at the same rate. The checking of this statement by considering

$$\begin{aligned} L = U^{A}(X^{A}, Y^{A}, S) &+ \lambda_1 [U^{B}(X^{B}, Y^{B}, S) - Z] \\ &+ \lambda_2 [X(K^{X}, L^{X}, S^{X}) - X^{A} - X^{B}] \\ &+ \lambda_3 [Y(K^{Y}, L^{Y}, S^{Y}) - Y^{A} - Y^{B}] \\ &+ \lambda_4 [K^{T} - K^{X} - K^{Y}] \\ &+ \lambda_5 [L^{T} - L^{X} - L^{Y}] \\ &+ \lambda_6 [S - S^{X} - S^{Y}] \end{aligned}$$

is left to the reader as an exercise. The result also applies where total emissions adversely affect production as well as having utility impacts – consider

$$\begin{aligned} L = U^{A}(X^{A}, Y^{A}, S) &+ \lambda_1 [U^{B}(X^{B}, Y^{B}, S) - Z] \\ &+ \lambda_2 [X(K^{X}, L^{X}, S^{X}, S) - X^{A} - X^{B}] \\ &+ \lambda_3 [Y(K^{Y}, L^{Y}, S^{Y}, S) - Y^{A} - Y^{B}] \\ &+ \lambda_4 [K^{T} - K^{X} - K^{Y}] \\ &+ \lambda_5 [L^{T} - L^{X} - L^{Y}] \\ &+ \lambda_6 [S - S^{X} - S^{Y}] \end{aligned}$$

where $\partial X/\partial S < 0$ and $\partial Y/\partial S < 0$.