The Equal Marginal Value Principle: A Graphical Analysis with Environmental Applications

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The typical undergraduate intermediate microeconomics textbook uses geometric arguments accompanied by two-dimensional diagrams to present the principles of cost minimization and profit maximization (see Pindyck and Rubinfeld 1995). These tools are sufficient to explain the behavior of a firm with one choice variable—for example, the level of production of a single product from a single factory. More realistic models, where firms have multiple products, factories, and technologies, are usually not presented because they require the use of more sophisticated mathematical techniques, such as Lagrange multipliers.

In this article, I develop a simple diagrammatic exposition of optimization principles for problems with two choice variables. In the basic model, the choice variables correspond to production levels in two different factories.[1] In addition to illustrating the familiar equal marginal value principle, the basic model makes more advanced topics, such as corner conditions and the role of convexity in optimization, accessible to a wide variety of students. An extension of the model illustrates the fundamental relationship between profit maximization and cost minimization. Other extensions are used to analyze important environmental issues, such as the cost effectiveness of policy proposals to reduce auto emissions, the economics of garbage disposal, and the economics of pollution abatement.

THE MODEL

Consider a firm that produces a single product in two different factories. Let q_1 be the quantity produced in the first factory and q_2 be the quantity produced in the second. The factories have cost functions $C_1(q_1)$ and $C_2(q_2)$ and associated marginal cost functions $MC_1(q_1)$ and $MC_2(q_2)$, respectively.[2] The firm wants to produce q_t units of the product so that the total cost of production is minimized. The solution may be found graphically by placing the marginal costs curves back-to-back so that they share the same y axis, but quantity produced increases in opposite directions along the x axis. The total cost associated with any particular division q_1^p, q_2^p of production between the two factories is found by placing a "ruler" of length q_t along the x axis so that the right edge is directly under q_1^p and then calculating the area bounded by the marginal cost curves and the ruler.

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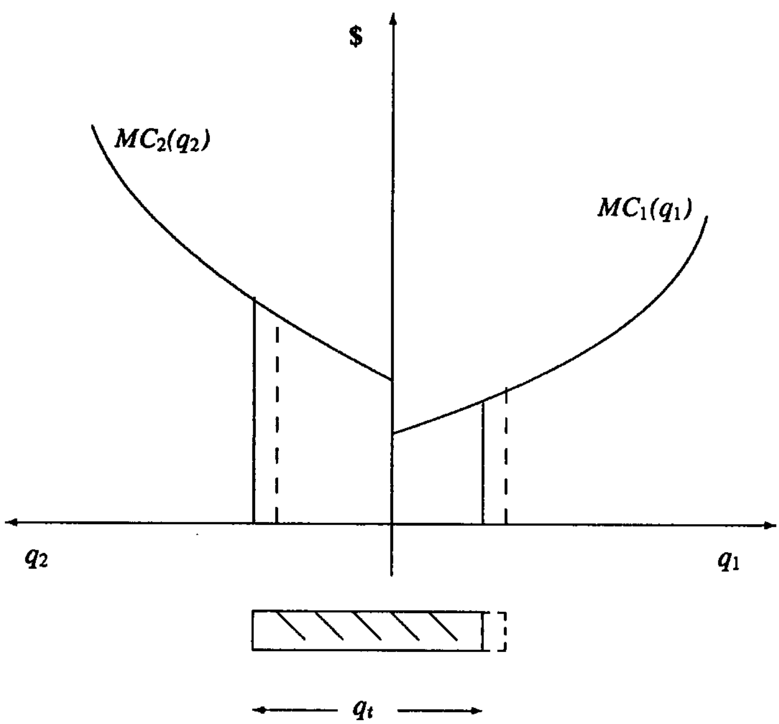
The optimal division of production that minimizes the cost of producing q_t can be found by sliding the ruler along the x axis until the bounded area is the smallest. This simple procedure nicely illustrates the equal marginal value principle in the "normal" case where the convexity of the cost curves leads to an interior solution. In Figure 1(a), the ruler is not at the optimal place. Sliding it a little to the right reduces the area under the second marginal cost curve more than it increases the area under the first. Hence, cost is minimized only when the marginal cost of production is equalized across the two factories. In other words, the marginal change in value associated with the first choice variable must be equal to the marginal change in value associated with the second choice variable.

Now suppose that the desired production level q_t decreases. Because the length of the ruler is equal to q_t, one uses a shorter ruler to find the optimal production plan. If q_t is very small, the optimal production plan calls for zero production in the second factory. Other types of corner conditions can be illustrated in a similar manner. In Figure 1(b), there is no point where marginal costs are equal; cost is minimized by producing everything in the second factory. In Figure 1(c), it is possible, but not optimal, to equate marginal costs. Sliding the ruler a little to the

FIGURE 1a
The Sliding Ruler



right reduces the total cost, and, hence, cost minimization dictates that production should be zero in the second factory.

The profit-maximizing behavior of a price-taking firm can be illustrated by placing a horizontal line through Figure 1(a) at a height corresponding to the market price of the product. The firm now *chooses* the total amount of production q_t at which profit is maximized. In other words, it selects the size of the ruler and places the ruler along the x axis so that the area between the price line and the marginal cost curves is as large as possible. Notice that this process ensures that the production levels will be chosen in a manner that minimizes cost. Thus, profit maximization implies cost minimization. In normal cases, price should be equal to marginal cost in both factories.

One can also consider more complicated joint profit-maximization problems. Suppose that the firm produces *different* products in the two factories and that an inventory constraint limits total production to q_t. (Alternatively, total production could be limited by an environmental standard.) Let p_i be the market price of the ith product and $MP_i(q_i) = p_i - MC_i(q_i)$ be the marginal profit from factory i when production is q_i. In this case, the marginal cost curves of Figure 1 are replaced

FIGURE 1b
Equal Marginal Costs Not Possible

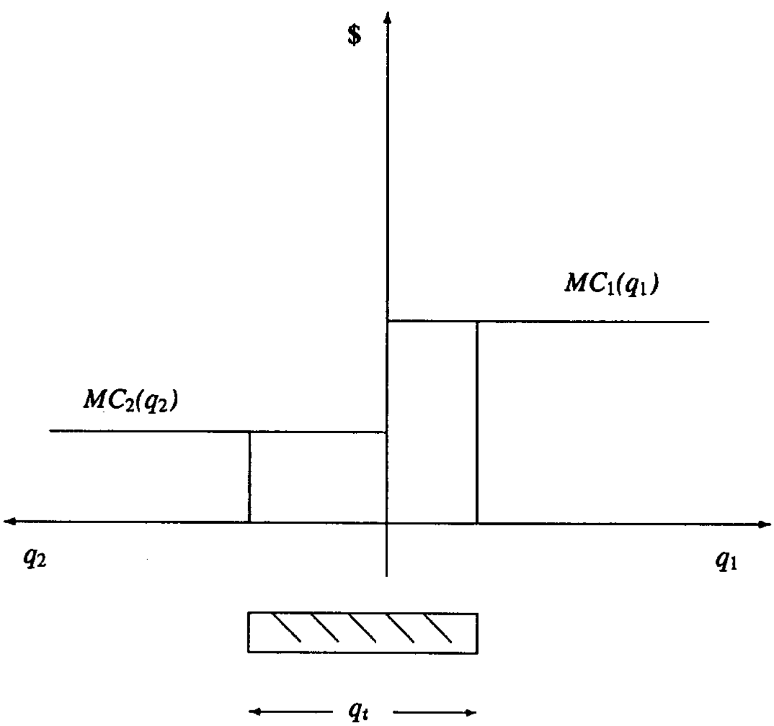
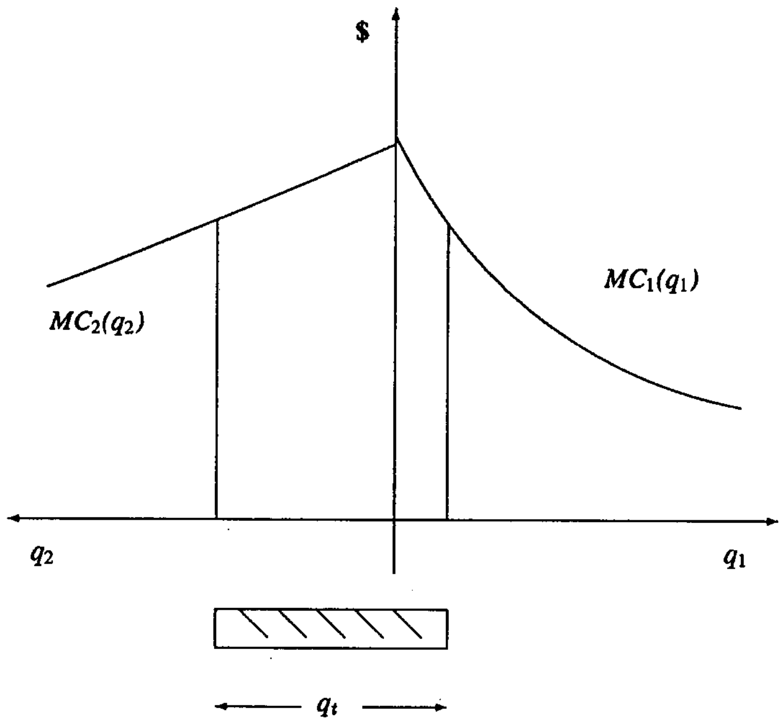


FIGURE 1c
Equal Marginal Costs Optimal



with the marginal profit curves. The firm places the ruler so that the area under the marginal profit curves is a maximum. The equal marginal value principle implies that the marginal profit in the two factories should be the same.

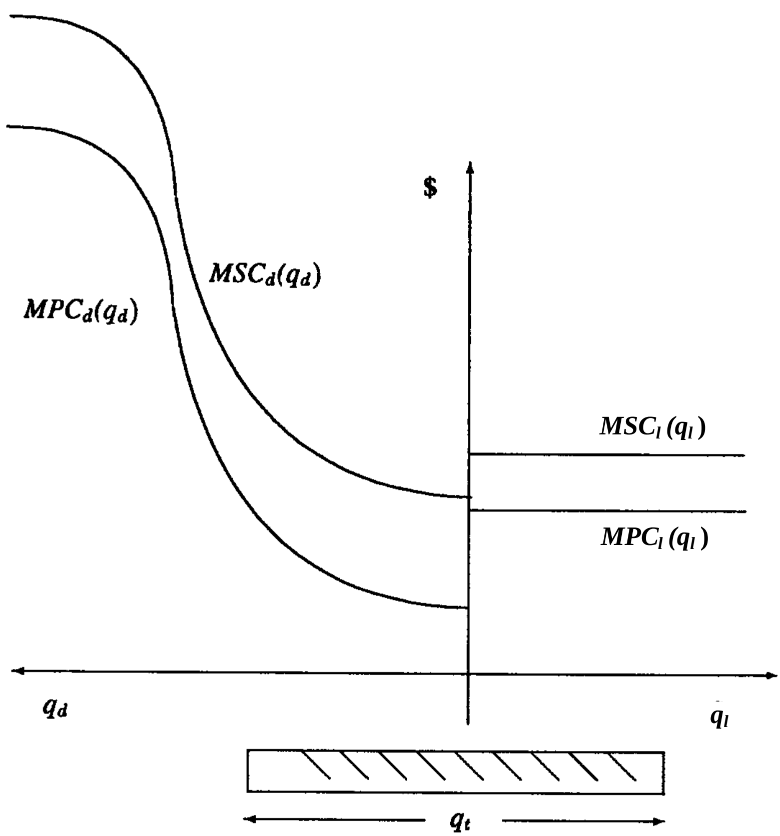
ENVIRONMENTALAPPLICATIONS

The sliding ruler method can easily be extended to applied microeconomics fields such as environmental economics. Consider the problem of finding the least-cost method of reducing automobile emissions by a given amount. Suppose that society can reduce emissions by making improvements to existing gasoline engines, mandating the use of electric cars, or employing a combination of both strategies. In this case, we interpret q_1 and q_2 as the amount of emissions reduced by gasoline engine improvements and electric car use, respectively. The marginal cost curves reflect the marginal cost to society of reducing emissions by these strategies. Students can sketch what they think the two marginal costs curves look like and then evaluate the cost effectiveness of current policy proposals by using the methods described above.

Now consider a model for garbage disposal.[3] A consumer can dispose of her garbage legally by taking it to a landfill. Alternatively, the consumer can use illegal methods such as dumping it in a vacant lot or burning it. The consumer must pay a fixed fee per unit of garbage taken to a landfill. Illegal disposal is free, but there is a probability that the consumer will be caught and have to pay a large fine per unit of garbage disposed of illegally. In addition to these private costs, both types of disposal may impose external costs on society. I assume that external costs associated with illegal disposal are more severe than those associated with legal disposal.

Suppose that the consumer wants to minimize the total private costs of disposing q_t units of garbage. Let q_d be the amount of garbage dumped illegally and let q_l be the amount of garbage taken to a landfill. The relevant marginal cost curves for the consumer and for society are shown in Figure 2. The consumer has

FIGURE 2
Disposal Costs for the Individual (*MPC*) and Society (*MSC*)



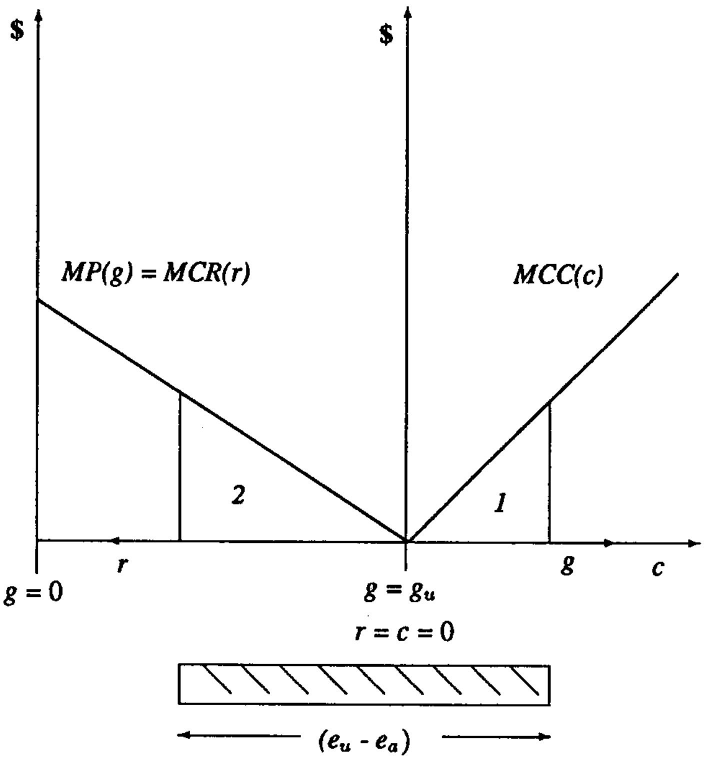
a constant marginal private cost of legal disposal (MPC_l) and an increasing marginal private cost of illegal disposal (MPC_d). (As the amount of illegal garbage disposal increases, the probability of getting caught increases.) The marginal social cost curves of legal (MSC_l) and illegal (MSC_d) garbage disposal are equal to the sum of the marginal private costs and any marginal external costs. The sliding ruler is in the optimal position for the consumer in Figure 2. Society, however, would be better off if more garbage was disposed of legally. It may be possible to align the interests of the consumer and society by *subsidizing* legal garbage disposal. If the subsidy does not change the consumer's decision about the total amount of garbage to be disposed, then it simply lowers the marginal cost of legal disposal. Thus the optimal position for the ruler shifts to the right.

General treatments of the economics of pollution use an external cost curve, an abatement cost curve, and the associated marginal curves to analyze policy proposals such as standards, taxes, or permit markets. I concentrate here on the derivation of the abatement cost curve. Consider a price-taking firm that is facing a pollution constraint. The firm is allowed to emit only e_a units of pollution into the environment. The abatement cost associated with e_a is equal to the total cost to a firm of the *least-cost method* of reducing the level of pollution emissions to e_a. The least-cost proviso is important, because a firm may have several options available to eliminate emissions. For example, the firm may reduce output, install pollution-control equipment, or use a combination of both strategies. In textbook presentations, the solution to the problem of selecting the least-cost method is usually not discussed.[4]

The abatement cost curve can be derived by using a modified sliding ruler diagram. Suppose that the firm produces one unit of pollution as a by-product of each unit of production. Let g be the amount of pollution generated by production, and let c be the amount of pollution eliminated by control technology. The amount of pollution emitted into the environment (e) is equal to the difference between the amount generated and the amount eliminated ($e = g - c$). Let $MP(g)$ be the marginal profit of the firm as a function of g and let $MCC(c)$ be the marginal pollution control cost as a function of c. An unconstrained firm would produce g_u units of pollution, where $MP(g_u) = 0$. Thus, $e_u = g_u$ units of pollution would be emitted into the environment. Because it faces the emission constraint, the firm wants to find the least-cost way of reducing emitted pollution from e_u to e_a. One could use c and g as the choice variables in this problem. But to employ the sliding ruler in the most direct manner, it is convenient to replace g with r, where r is defined as the amount of pollution eliminated by reducing output below g_u. Hence, the two relevant marginal cost curves are the marginal cost of pollution control and the marginal cost of output reduction.

The graphical solution to the problem is found by extending the sliding ruler diagram to include the relationship between g and r. Figure 3 illustrates the case in which MP is linear and decreasing and MCC is linear and increasing. As before, the marginal cost curves are placed back-to-back so that they share the same y axis and the values of the choice variables increase in opposite directions along the x axis. The marginal profit curve is superimposed on this structure. The variable g increases along the x axis in a direction opposite to r. In addition, the

FIGURE 3
Pollution Elimination and Output Reduction



origins of the g and r coordinate systems are separated by a distance g_u. The marginal cost of output reduction, denoted by $MCR(r)$, is equal to the forgone marginal profit. The length of the sliding ruler is equal to $e_u - e_a$, the amount of emissions that must be eliminated. The ruler is in the optimal location when the forgone marginal profit is equal to the marginal cost of pollution control. As the emission constraint becomes more restrictive (e_a decreases), the firm will simultaneously decrease output and increase pollution control to retain the equal marginal value condition.This is a powerful comparative statics result, obtained without the usual trappings of differential calculus.

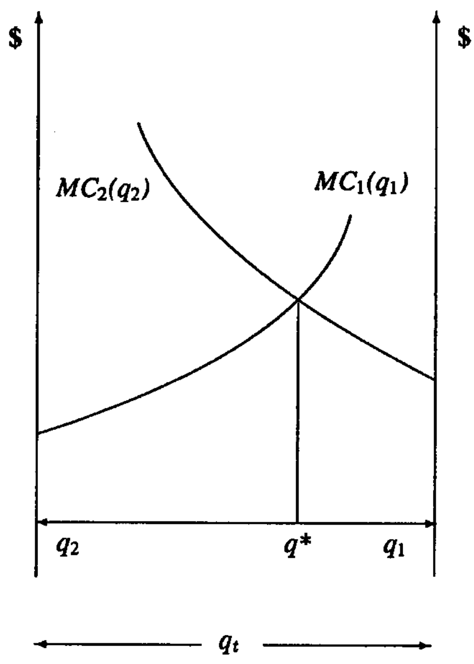
Having solved the firm's problem, one can now calculate the abatement cost. The area of triangle 1 in Figure 3 corresponds to the total-pollution control cost to the firm of meeting the emission constraint. Likewise, the area of triangle 2 corresponds to the total lost profit resulting from output reduction. The sum of these two areas is equal to the abatement cost associated with e_a. The entire abatement cost curve can be calculated by repeating the procedure for various values of e_a. Notice that the marginal abatement cost associated with a particular e_a is equal to the height of either triangle 1 or triangle 2 in Figure 3. As the constraint becomes stricter, the marginal abatement cost increases. In other words, reducing emitted pollution by an additional unit becomes increasingly costly to the firm.

AN ALTERNATIVE APPROACH

In the sliding ruler framework, the marginal cost curves are oriented away from each other but share the same *y* axis. Alternatively, one could orient them toward each other and separate the *y* axes by a distance equal to the magnitude of the constraint.[5] Figure 4 illustrates this approach for the two-factory cost-minimization problem. Any point on the *x* axis represents a division of q_t units of production between the two plants. The optimal division (q^*) corresponds to the *x* coordinate of the point of intersection of the two marginal cost curves.

Figure 4 can be modified to illustrate many of the examples presented in this article. But there is an important difference between the sliding ruler framework and this alternative approach. Suppose that one wants to discuss the comparative statics of a change in the magnitude of the constraint. In the sliding ruler framework, only the length and position of the ruler need to be adjusted; the marginal cost curves remain fixed. Conversely, in the alternative approach, a change in the constraint requires that *both* marginal cost curves be adjusted. For example, con-

FIGURE 4
An Alternative Diagram



sider the comparative statics of a change in desired production in the basic two-factory problem. One can use Figure 1(a) to illustrate the change in the solution by placing a slightly smaller ruler over the existing one. Production levels in both plants decrease. It is more difficult to illustrate the same conclusion with Figure 4. Here the distance between the *y* axes must be reduced. Suppose that the position of only one axis is adjusted, so that $MC(q_2)$ shifts to the left. It is not obvious that the amount of production in the second plant has decreased. One must redraw both marginal cost curves so that they are separated by a slightly smaller amount, and the *x* coordinate of the point of intersection is the same as before. Then it is clear that production levels in both plants decrease.

CONCLUSION

The equal marginal value principle for allocating resources among multiple processes is a powerful element of the economist's toolkit. The techniques presented here can be used to explain the equal marginal value principle to students in a simple way. Perhaps more important, it is also easy to explain why exceptions to the equal marginal value principle occur. Such exceptions are often important for policy analysis. In the automobile pollution example, society may be better off if pollution reduction efforts are concentrated on one particular strategy. In addition to the problems considered here, other applications include power generation, where one can analyze the choice between nuclear and fossil fuels,and risk reduction, where one can study how to allocate resources to reduce risk from several sources.

NOTES

1. Frank (1994) presents a simplified analysis of a similar two-factory problem.
2. For simplicity, all cost curves are assumed to be long-run curves,and all marginal cost curves are assumed to be either nondecreasing, nonincreasing, or constant.
3. This example illustrates some of the basic insights found in Fullerton and Kinnaman (1995), although their model is slightly different.
4. See Tietenberg (1992) as well as Callan and Thomas (1995). Pearce and Turner (1990) present what appears to be an incorrect solution,although it is difficult to determine the exact features of their model.
5. For an example of this approach in an environmental context, see McInerney (1976).

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