

Natural resource economics under the rule of Hotelling

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Abstract. Natural resource stocks held in situ are physical assets. Equilibrium in the assets market requires that their rates of return be such that their owners are just willing to hold on to them rather than invest elsewhere. I discuss a number of factors relating to the evolution of extraction costs, to the durability of the resource, to market structure, and to uncertainty, that are important in correctly characterizing the rate of return on holding exhaustible natural resource stocks. The emphasis is put on how those factors can potentially help bridge the gap between the basic Hotelling's rule of natural resource exploitation and the historical behaviour of the flow price of a number of resources. I also highlight some theoretical and empirical issues that need further attention.

L'économie des ressources naturelles à la lumière de la règle d'Hotelling. Les stocks de ressources naturelles sont des actifs dont le taux de rendement doit, à l'équilibre, être égal à ce que leur propriétaire pourrait obtenir comme rendement en les plaçant ailleurs. Je discute de facteurs, reliés à l'évolution des coûts d'extraction, à la durabilité de la ressource, à la structure des marchés et à l'incertitude, qui sont importants pour caractériser correctement le taux de rendement sur la détention de stocks de ressources naturelles. L'accent est mis sur la capacité de ces facteurs à réconcilier la règle d'Hotelling élémentaire avec le comportement historique des prix des ressources naturelles tel qu'observé sur les marchés des flux de ressources. J'attire également l'attention sur des questions théoriques et empiriques qui mériteraient qu'on s'y attarde.

Presidential address delivered at the 41st Annual Meeting of the Canadian Economics Association, Halifax, 2 June 2007. I would like to thank Didier Tatoutchoup for his assistance in collecting the data and Isabelle Boudreau for bibliographic assistance. Very special thanks also to my friends and long-time collaborators Pierre Lasserre, Ngo Van Long, Michel Moreaux, and Stephen Salant for their comments and especially for the many enjoyable and profitable discussions about natural resource economics we have had over the years. Email: gerard.gaudet@umontreal.ca

1. Introduction

The year 2006 marked the 75th anniversary of the publication in the *Journal of Political Economy* of Harold Hotelling's 'The economics of exhaustible resources' (Hotelling 1931). The word *seminal* takes all its sense when we write or speak about this article. Rarely has a single paper had as much influence on the development of a whole field of economic research as this one has had on the economics of exhaustible resources.¹ This anniversary provides a good occasion to look back at how the famous Hotelling's rule of natural resource exploitation has evolved as a framework for understanding the functioning of natural resource markets and to look forward at some unresolved issues, both theoretical and empirical, that would help reconcile the theory and the facts about resource price behaviour over time.

Hotelling's motivation in writing that paper was twofold. He was reacting to the demand for regulation of the exploitation of exhaustible natural resources by the Conservation Movement, which had been particularly strong in the U.S. during the period 1890 to 1920. But he also wanted to propose an economic theory of exhaustible resources to remedy the inadequacy of what he called the 'static-equilibrium type of economic theory' for analyzing an inherently dynamic problem. His analytical approach to the problem of the efficient use of exhaustible resources was similar to that which he had used in a wonderful paper published in 1925 in the *Journal of the American Statistical Association*, titled 'A general mathematical theory of depreciation' (Hotelling 1925). In that paper, he introduced the definition of the depreciation of an asset as the decrease in the discounted value of its future returns. Not only is the topic related to that of the depletion of a natural resource, but it required the use of the calculus of variations, as did his economics of exhaustible resources.

Hotelling's treatment of the economics of exhaustible resources did not get serious attention before the 1970s. There are probably two explanations for this. The first is that economists, even academic economists, read the newspapers, and the newspapers' attention was directed to other more pressing issues in the middle of the Great Depression, as it was during most of the war years that followed in the 1940s.²

The adequacy of natural resources for sustained economic growth became a concern after World War II, particularly in the U.S. This concern gave birth to the President's Materials Policy Commission (Paley Commission), whose report was made public in 1952, and, as an outgrowth, to Resources for the Future, the

1 By exhaustible resources Hotelling means non-renewable resources, as opposed to renewable resources. The expression 'exhaustible resource' is somewhat of a misnomer, since both types of resource are exhaustible. Hotelling recognizes that 'a complete study of the subject would include semi-replaceable assets such as forests and stocks of fish' (renewable resources), but chooses to limit himself to 'absolutely irreplaceable assets' (non-renewable resources) (Hotelling 1931, 139-40).

2 The same volume of the *Journal of Political Economy* contained a paper titled 'Some theoretical aspects of stock market speculation' and another titled 'Savings, investment, and the control of business cycles,' which were certainly more topical at the time.

influential Washington think tank. Hotelling's contribution nonetheless went unnoticed throughout the 1950s. Resources for the Future sponsored the publication in 1963 of *Scarcity and Growth*, by Harold Barnett and Chandler Morse (Barnett and Morse 1963), which was the first systematic analysis of long-run scarcity measures of a number of natural resources. Barnett and Morse make just a passing reference to Hotelling's 1931 paper.³ So lack of public concern for the issues of natural resource use is not sufficient to explain the lack of interest for Hotelling's contribution before the 1970s.

The other reason why the paper did not get the early attention it deserved is that it was a difficult paper, given the level of mathematical sophistication of the economic profession at the time. In fact, according to Hotelling's own later account, his paper was rejected by the *Economic Journal* because its mathematics were too difficult (Arrow 1987; Darnell 1988).

In the 1970s another period of intense public concern for natural resource scarcity followed. This was triggered by *Limits to Growth*, the report for the Club of Rome published in 1972 by Dennis Meadows and his co-authors (Meadows et al. 1972), predicting catastrophic consequences for the early 21st century unless economic growth was seriously cut back, and by the 1973 oil embargo and the ensuing energy crisis. This time academic economists were ready to tackle the analytical framework put forth by Hotelling in 1931, being equipped with the optimal control theory learned by working on optimal economic growth problems in the 1960s. William Nordhaus published in 1973 an important study entitled *The Allocation of Energy Resources*, which applies the Hotelling model to the problem of how different energy sources should be used (Nordhaus 1973). He also published the next year a critical analysis of *Limits to Growth* in the *American Economic Review* (Nordhaus 1974). Robert Solow devoted his 1974 Ely Lecture to the American Economic Association to the subject of natural resource economics (Solow 1974). The same year the *Review of Economic Studies* published a symposium devoted to the topic of exhaustible resources, with contributions by Partha Dasgupta, Geoffrey Heal, Tjalling Koopmans, Joseph Stiglitz, and Robert Solow, among others. Natural resource economics began to acquire greater prominence as a field of research. The now famous Hotelling's rule was at the heart of it and it still is.⁴

2. Hotelling's rule

Like the stock of any physical capital, a stock of natural resource *in situ* is an asset to its owner. In a market economy, the value of this asset, like for any other

3 They characterize Hotelling's paper as 'contribut[ing] usefully in a *technical way* [my emphasis] to the question of whether, and why, the time distribution of resource use and investment under private enterprise conditions may be socially erroneous' (Barnett and Morse 1963, 46)

4 That is not to say that there were no significant contributions to the natural resource literature before the 1970s. To mention a few: Gray (1914), Scott (1955b), Gordon (1967), and Herfindahl (1967), on non-renewable resources; Gordon (1954), Scott (1955a), and Smith (1968) on renewable resources.

capital asset, will be related to the rate of return it is expected to yield to its owners.

Typically, the rate of return on a physical asset can be decomposed into three components:

- A first component is attributable to the flow of product generated by the marginal unit of the asset – its rate of marginal productivity or dividend rate.
- A second component is due to the fact that the asset's physical characteristics may change over time, a factor that may or may not depend on the use being made of it or on the size of the stock being held.
- A third component is the rate at which the asset's market value changes over time. This may be negative, as long as it is offset by some other positive components of the return.

In order for the asset markets to be in equilibrium, this rate of return must be equal to the rate of return that can be expected by its owners if they were to sell the marginal unit and invest the proceeds elsewhere. I will assume for now that the alternative is a 'bond' that yields an exogenously given riskless rate of return and will call it the 'rate of interest.'

In the case of some physical asset such as buildings, machinery, and equipment, the first component usually refers to the marginal product derived from its use as an input in the production process. The second component refers to the rate of physical depreciation of the asset, which enters negatively in the rate of return. It may be due to wear and tear from the use of the asset or simply due to obsolescence and is often treated as a constant in theoretical work. The third component refers to the capital gains that can be had by holding this asset.

Suppose, now, the asset in question is a non-renewable resource, such as a mineral deposit or a stock of oil in the ground. Such assets, being non-reproducible, have the property that the size of their existing stock cannot be increased over time. Furthermore, holding such an asset *in situ* yields no dividend: as long as it is left in the ground, it is totally unproductive, contrary to a machine or a piece of equipment, which are capable of generating a flow of services. Hence, the first component is identically zero. As for the second component, there is usually no exact equivalent in the case of resource stocks, in the sense that physical deterioration will not occur from simply holding the asset in the ground. There is a sense, however, in which keeping the marginal unit of the asset stored in the ground rather than extracting it *prevents* the average quality of the remaining stock from deteriorating. This second component therefore enters *positively* into the rate of return, rather than negatively. Let me also set this component to zero for now and come back to it later. This leaves the rate of appreciation in value as the only source of return on the stock of natural resource.

The value of the marginal unit of resource held in the ground is what it can fetch on the flow market, net of the cost of taking it out of the ground. If $p(t)$ is the current *flow price* the resource can fetch on the market once extracted and

$c(t)$ is the marginal cost of extracting it at date t , then its marginal value in the ground must be

$$\pi(t) = p(t) - c(t),$$

which represents the *asset price* of the resource.

If the rate of interest is denoted r , then asset markets equilibrium requires

$$\frac{\dot{\pi}(t)}{\pi(t)} = r. \quad (1)$$

This is the famous Hotelling's rule, derived as an equilibrium condition on the asset markets. It states that the *net price* of the natural resource – its asset price – must grow at the rate of interest.

If the marginal cost of extracting the resource is independent of the rate of extraction and invariant over time, then this immediately yields a prediction as to the behavior of the market price over time, namely,

$$\frac{\dot{p}(t)}{p(t)} = r \left(1 - \frac{c}{p(t)} \right). \quad (2)$$

If this were a correct representation of reality, we should observe the price of non-renewable resources continuously growing at a rate that tends towards the rate of interest as the share of cost in price gets smaller and smaller over time and that of the scarcity rent gets higher and higher.

This is not what we have been observing over the last century and more. Using U.S. price data for the period 1870–2004 for copper, lead, zinc, coal and petroleum, 1880–2004 for tin, 1900–2004 for aluminium and nickel and 1920–2004 for natural gas, I have plotted in figures 1 to 10 the rate of change of price of each of those seven non-renewable minerals and three non-renewable fossil fuels. The first thing that we notice is the high volatility in the rate of change of those prices. But more significantly for the purpose at hand is the fact that this volatility appears centred at zero. In fact, in none of the 10 cases is the mean rate of change of price significantly different from zero.⁵ It is very hard to detect any trend in the actual price levels of those resources and certainly not the kind of positive trend that is suggested by the above statement of Hotelling's rule.⁶

Is this sufficient to invalidate Hotelling's rule? I believe not. When viewed as an asset markets equilibrium condition, this observed discrepancy means that in reality there must be more to the return on holding non-renewable natural resource

5 The p-values range from 0.82 to 0.98.

6 The analysis by Berck and Roberts (1996) of the actual prices of nine non-renewable natural resources up to 1991 (the same resources as in figures 1 to 10, plus iron and minus nickel and tin) using time-series methods leads them to conclude that there is at best 'a weak supposition that natural resource prices will rise' (Berck and Roberts 1996, 65). A quick glance at the evolution of those prices between 1991 and 2005 leads one to conjecture that adding those fourteen years of observations would make the supposition weaker still.

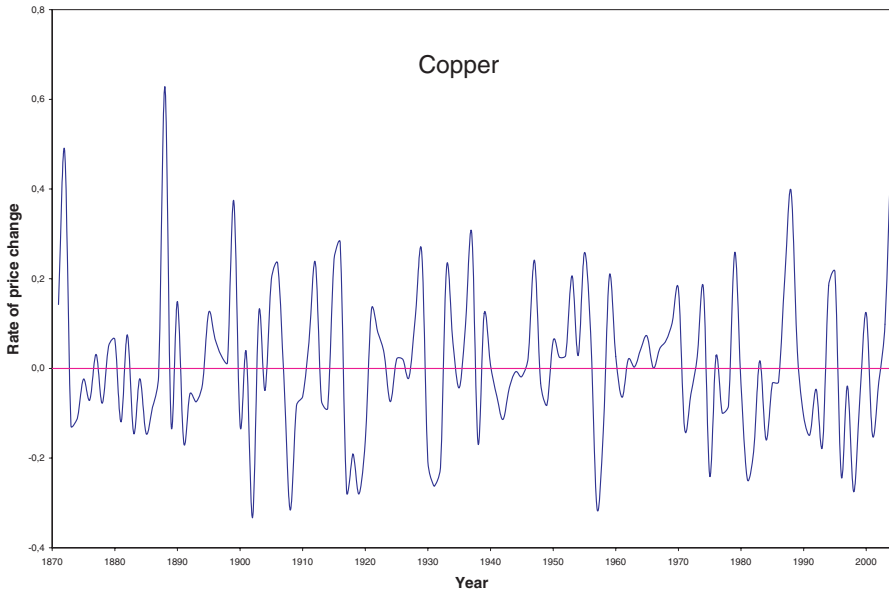


FIGURE 1 Rate of change of the U.S. price of copper, 1870–2004

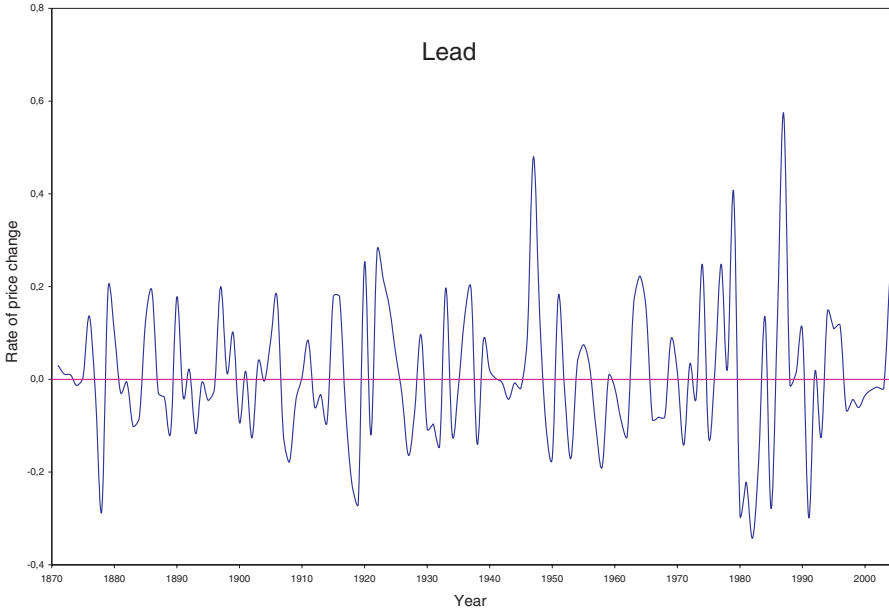


FIGURE 2 Rate of change of the U.S. price of lead, 1870–2004

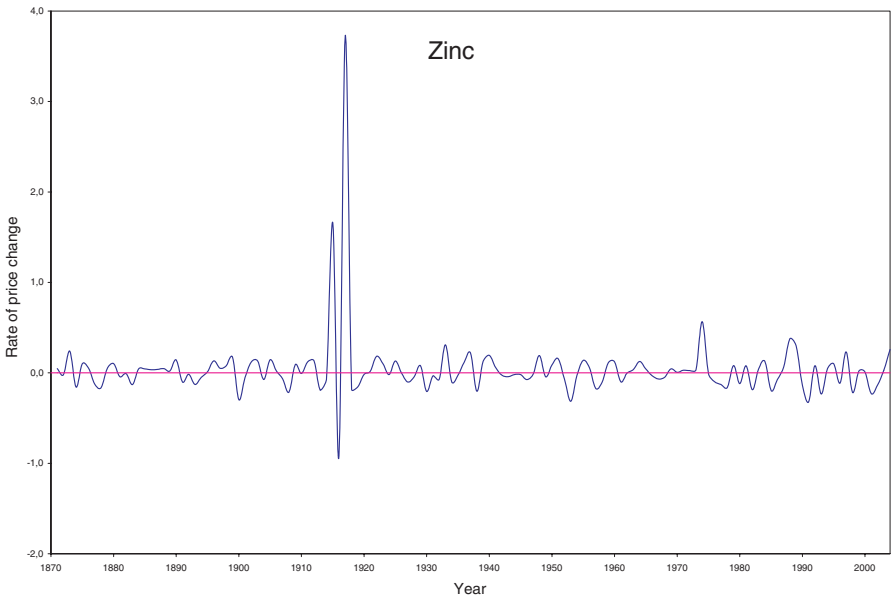


FIGURE 3 Rate of change of the U.S. price of zinc, 1870–2004

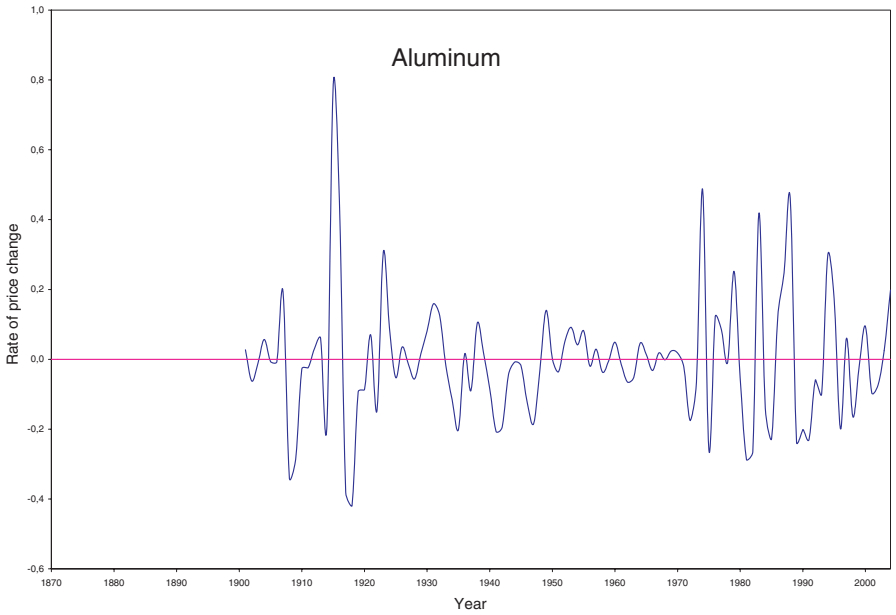


FIGURE 4 Rate of change of the U.S. price of aluminium, 1900–2004

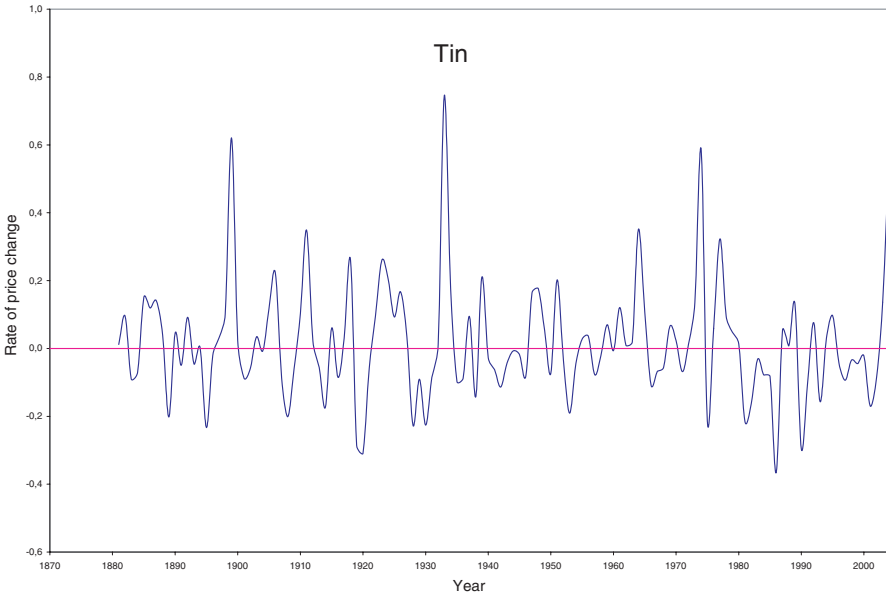


FIGURE 5 Rate of change of the U.S. price of tin, 1880–2004

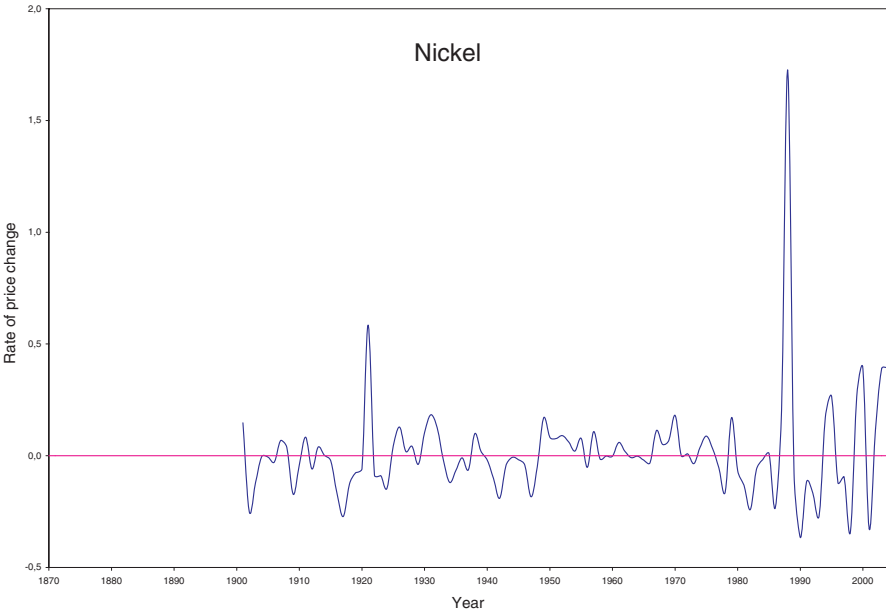


FIGURE 6 Rate of change of the U.S. price of nickel, 1900–2004

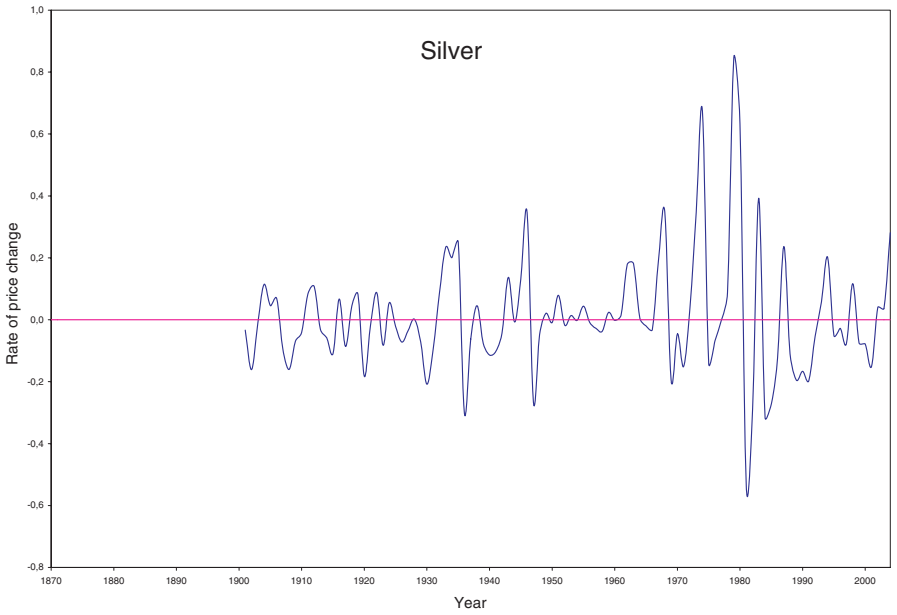


FIGURE 7 Rate of change of the U.S. price of silver, 1900–2004

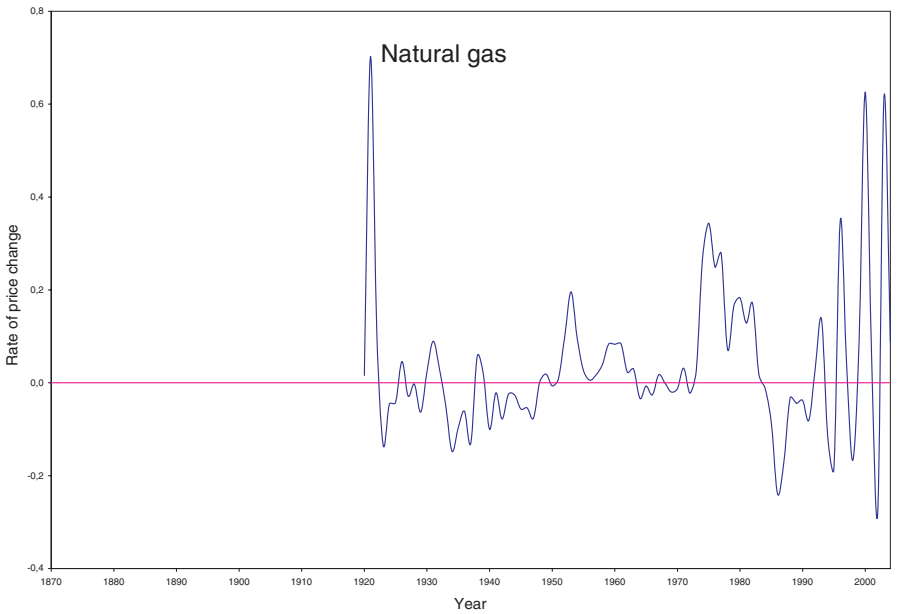


FIGURE 8 Rate of change of the U.S. price of natural gas, 1920–2004

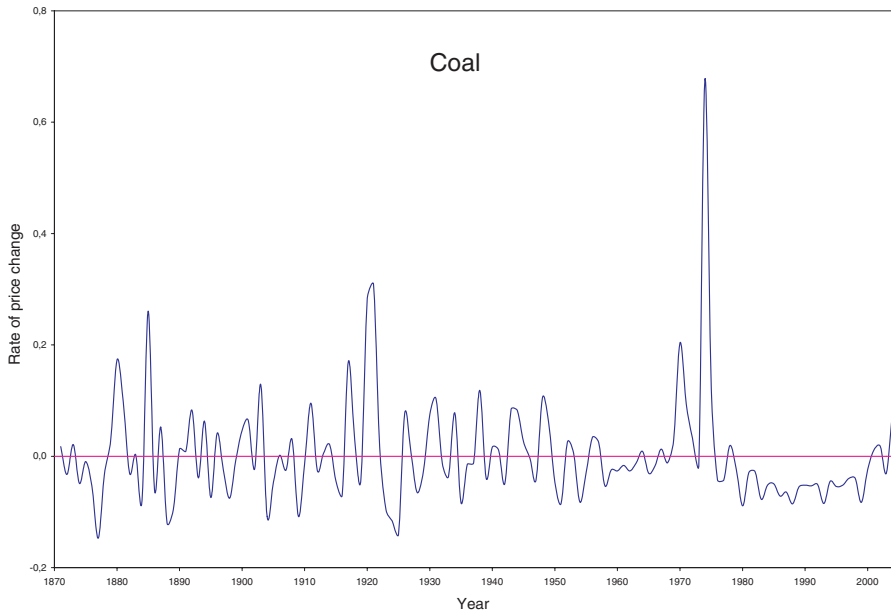


FIGURE 9 Rate of change of the U.S. price of coal, 1870–2004

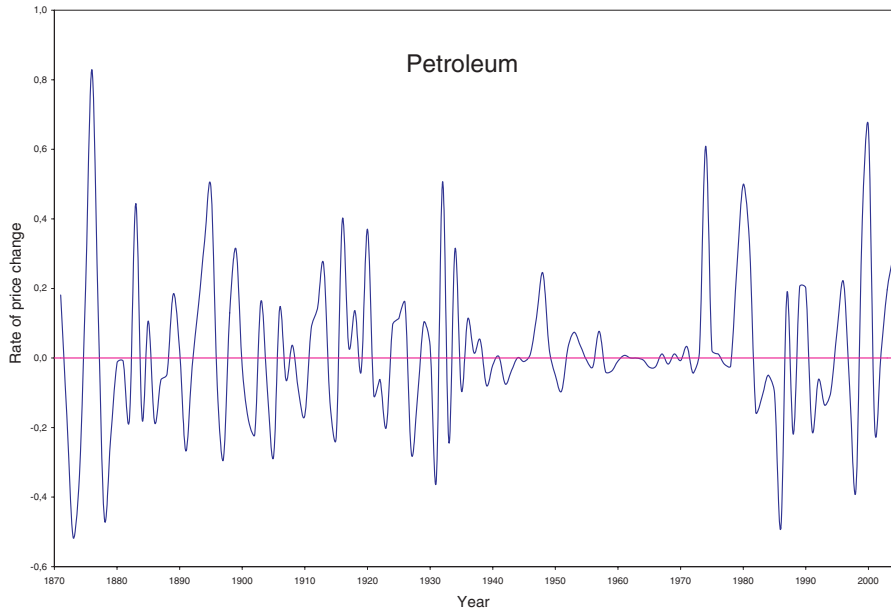


FIGURE 10 Rate of change of the U.S. price of petroleum, 1870–2004

stocks than simply the rate of price appreciation. Factors related to extraction costs, to durability of the resource, to market structure and to uncertainty must also be taken into account in order to correctly characterize the rate of return on resources. In what follows I will discuss in turn how each of those factors could potentially help to bridge the gap between the underlying theory and the evidence.

3. Costs

The first factor that comes to mind that is likely to attenuate the growth in the flow price of the resource is a decreasing cost of extraction due to technological progress. Technological change was an important factor in the finding of Barnett and Morse that, except possibly for forestry, the *average* cost of extractive resources had been following a declining trend over the period of close to 100 years they were considering.

For the sake of the argument, write the unit cost of extraction as $c(t) = ce^{-\alpha t}$, to reflect exogenous technological progress at the rate α in the extraction technology. Then Hotelling's rule yields the following prediction for the rate of change in the market price $p(t)$:

$$\frac{\dot{p}(t)}{p(t)} = r \left(1 - \frac{c(t)}{p(t)} \right) - \alpha \frac{c(t)}{p(t)}. \quad (3)$$

The rate of change of the market price is thus the weighted sum of the rate of interest and the rate of decrease in cost due to technological change. If the share of marginal cost in price is sufficiently high, the effect of the rate of technological change on cost dominates. The reverse is true if the share of marginal cost in price is sufficiently low. This is consistent with a price path that would be at first decreasing and then increasing (Slade 1982). It cannot justify, however, forever flat or decreasing price paths. In the long run, as the share of cost in price becomes negligible, the effect of the rate of interest must come to dominate, and the rate of growth of price must eventually become positive and again approach the rate of interest over time.

There is another reason besides technological progress for the cost of extraction to vary over time. It can be interpreted as having to do with the second component of the rate of return on physical assets, which I have so far neglected. As the resource stock gets depleted, we can expect the marginal cost of extraction to increase, owing to the fact that the resource tends to be less easily accessible and of lower grade. A marginal addition to current resource extraction not only uses up the resource stocks, but it uses up the cheapest available and hence increases all future costs. This Ricardian effect has been called a 'degradation cost' (Solow and Wan 1976) and can be assimilated to physical depreciation that can be prevented by not depleting the asset. To capture it, write the marginal cost of extraction as

a decreasing function of the remaining stock, in addition to rate of technological progress: $c(t, X(t)) = e^{-\alpha t} f(X(t))$, $f'(X(t)) < 0$.⁷ Hotelling's rule must then be modified to reflect this stock effect. It becomes

$$\frac{\dot{\pi}(t)}{\pi(t)} - \frac{e^{-\alpha t} f'(X(t))x(t)}{\pi(t)} = r, \quad (4)$$

where $x(t)$ represents the rate at which the stock $X(t)$ is being depleted. Thus, leaving the marginal unit of resource *in situ* provides an additional return, in the form of a lower cost of extraction in the future. Hotelling's rule now dictates that the net equilibrium price grows at a rate that is less than the rate of interest. But since this stock effect directly enters the net price through its effect on cost, the expression for the rate of growth of the flow price remains unchanged, except for the fact that the level of marginal cost now depends on the remaining stock. Flow price behaviour remains qualitatively the same. The prediction is still that the rate of growth of the market price is the weighted sum of the rate of interest and the rate of decrease of extraction cost due to technological progress. Unless extraction becomes unprofitable before the stocks are exhausted, we should still observe that price will eventually be growing at a rate that approaches the rate of interest as the share of cost in price becomes smaller and smaller.

As just sketched, the depletion effect could be interpreted as being due strictly to physical reasons, as if lower-cost resources had to be exploited first in order to gain access to the higher-cost ones. This is often the case in any one resource pool. But at an aggregate level it is realistic to assume that many resource pools that differ in their cost of extraction are available for exploitation at any one time. Then it is economic efficiency, not nature, that dictates that, of any two resource deposits, the low-cost one should be depleted before moving on to the next highest-cost one (Herfindahl 1967; Solow and Wan 1976).⁸ As long as the stocks in both resource pools are positive, asset markets equilibrium requires that Hotelling's rule hold for each. Hence, the asset price of each must be growing at the rate of interest but with the asset price of the high-cost deposit lower than that of the low-cost deposit, since the flow price they can get on the market is the same for both deposits (I am assuming identical products). The owner of the high-cost deposit will find it unprofitable to extract and will hold on to his asset as long as the low-cost deposit is not depleted. When the switch to the higher-cost

7 For careful analyses of the effect on the predicted time path of the asset price (or scarcity rent) of other more general hypotheses concerning the extraction cost function, see Farzin (1992, 1995).

8 The problem of the efficient order of use of many resource pools has been generalized to resource pools that may exhibit non-convex cost of extraction over some range (Weitzman 1976), to resource pools that are differentiated by their set-up cost as well as their marginal cost (Hartwick, Kemp, and Long 1986) and to a general equilibrium context (Kemp and Long 1980; Amigues et al. 1998). It has also been generalized to resource pools that are differentiated by their geographical location with respect to many spatially distributed markets (Gaudet, Moreaux, and Salant 2001) and by characteristics which distinguish them in their efficacy in many different end uses (Chakravorty and Krulce 1994; Chakravorty, Roumasset, and Tse 1997).

deposit occurs, there will be a jump in the cost of extraction. Hence, the asset price will have grown on average at less than the rate of interest in the long run (Dasguspta and Heal 1979, 172–5). But the flow price itself will be continuously growing.

4. Durability

When we think of non-renewable resources, we often think of fossil fuels, such as oil, natural gas, and coal. But an often neglected fact is that many non-renewable resources – in particular, metals – are durable goods. In fact, of the 10 non-renewable resources whose price trend we just examined, seven can be considered durable resources, in the sense that they are not consumed in a single usage, as are fossil fuels.

Unlike fossil fuels and other non-durable resources, durable resources are capable of yielding a continuous flow of services once extracted. Thus, the demand for those resources is (at least partly) a stock demand. Once above ground, this stock of durable good becomes a conventional physical asset, and the asset markets equilibrium requires that its rate of return be equal to the rate of interest. This rate of return will include a dividend component (the flow of services it generates) and a physical depreciation component, in addition to a capital gains component, which corresponds to the rate of appreciation of the market price of the good. Hotelling's rule must still hold for the asset held below ground, which means that this market price net of the cost of extraction must also be growing at the rate of interest. But it does not follow anymore from Hotelling's rule that market price will necessarily be growing.

In fact, we can easily imagine a scenario where price would be always falling, even though the net price (the price of the asset in the ground) is always rising at the rate of interest (Levhari and Pindyck 1981). For this, simply assume that demand is stationary and that the resource is perfectly durable, so that there is no physical depreciation. Then, the long-run desired stock of the durable above ground is a constant. If marginal extraction cost is an increasing function of the rate of extraction, then this desired stock will be approached smoothly, at a decreasing rate. If the desired stock exceeds the available reserves, then the resource is scarce and its net price must follow Hotelling's rules. But since demand is a decreasing function of the stock already in circulation, market price must be decreasing.

Of course, in the real world demand is not stationary and the rate of physical depreciation is not zero and it may itself depend on price (Henderson et al. 2007). But even if we relax those assumptions, the price path can be expected to exhibit a decreasing phase before starting to increase. The rate of decrease and the length of the decreasing phase will depend on the rate of depreciation and the rate of growth of demand (Levhari and Pindyck 1981). Clearly, durability can be an important factor in reconciling Hotelling's rule with the long-run price paths we observe.

5. Market structure

Another factor which can influence the rate of return on resource stocks is the presence of imperfect competition.

If the resource stock is in the hands of a monopolist, the marginal value to the owner of the stock of resource left in the ground will be equal to the marginal profit it can bring on the flow market once extracted. To a monopolist, this is less than the net price. The asset markets equilibrium condition will still require that the rate of return on the resource stock be equal to the rate of interest. Only now the rate of appreciation of the *in situ* value is measured not by the rate of change of the net price, but by the rate of change of the monopolist's marginal profit. If we neglect changes in the marginal cost over time, the prediction is then that the rate of growth of price will be given by (Stiglitz 1976):

$$\frac{\dot{p}(t)}{p(t)} = r \left(1 - \frac{c}{\eta(t)p(t)} \right) - \frac{\dot{\eta}(t)}{\eta(t)}, \quad (5)$$

where $\eta(t)$ denotes the ratio of marginal revenue to price.

There are two effects of the introduction of a monopolist. The first comes from the fact that even if the proportion of marginal revenue to price does not vary over time (η does not change), price will be growing at a slower rate than in a competitive market simply because marginal revenue is smaller than price and hence the weight given to the rate of interest is smaller. Thus, even if the elasticity of demand remains unchanged over time, as would occur if demand were isoelastic and stationary, the price path will appear flatter than in a perfectly competitive world unless the marginal cost of extraction is negligible. The second effect comes from the fact that the gap between price and marginal revenue can be expected to change over time. If this gap is decreasing over time (η is growing), this will tend to further flatten the price path. The difference between price and marginal revenue being inversely related to the elasticity of demand, this will happen if the elasticity of demand is increasing over time. This is what is to be expected as substitutes for the resource are discovered and is therefore not an unrealistic assumption.

The absolute level of the price path will depend on the stock of the resource that is available for extraction. Since the flow market equilibrium requires that price be the same under monopoly and perfect competition at the date of exhaustion of the resource stock, this means that the monopoly price must initially be higher and eventually become lower than the competitive price. This reasoning assumes that the available stock of resource is the same under monopolistic and competitive industry structures.⁹ If costly efforts must be expended to determine the initial resource stock, then the monopolist may possibly further exert his monopoly

⁹ The fact that the monopolist's price is flatter than the perfectly competitive price path means that the monopolist will take a longer time than the competitive market to exhaust the same initial resource stock, hence the saying, 'the monopolist is the conservationist's best friend.'

power by choosing a lower initial stock than would price-takers (Gaudet and Lasserre 1988). However, this does not change the fact that monopoly power will tend to flatten the price path.¹⁰

I have so far discussed imperfect competition assuming a pure monopolist. As Hotelling himself remarked, a more realistic market structure for many non-renewable resources is some form of oligopolistic competition.¹¹ The flattening effect on the price path just described will still be felt, of course, since it is due strictly to the presence of some market power. However, an oligopolistic world is much more complex than either perfect competition or pure monopoly. It involves strategic interactions in an inherently dynamic context, since the resource stock evolves over time. The strategic considerations involved themselves may further modify the way the rate of return on holding the stock of resource is to be characterized, as we will see.

The pioneering works on oligopolistic markets for non-renewable resources were motivated by the post-1973 oil market and the presence of OPEC as a dominant player. Thus, the first approach to the problem (Salant 1976) treated the market as composed of a dominant cartel and a competitive fringe and adopted the Nash-Cournot equilibrium concept, whereby each player ignores the effects of his decisions on the strategies of his rivals. Under this approach, in solving for an equilibrium it is assumed that the competitive fringe takes as given the price path of the dominant player and chooses a production path, while the dominant firm takes as given the production path of the competitive fringe and chooses a price path. This type of market structure provided the framework for a number of subsequent analyses (for instance, Pindyck 1978; Salant 1982; Lewis and Schmalensee 1979; Ulph-Folie 1980).

A number of other studies have retained the cartel-versus-fringe assumption, but have treated the cartel as a Stackelberg leader (Gilbert 1978; Newbery 1981; Ulph 1982; Groot, Withagen, and de Zeeuw 1992). Under this approach the cartel is aware of the effect of its decisions on the decisions of the fringe and takes it into account. The fringe, as a follower, chooses a production path in reaction to the output path of the cartel. The cartel, as the leader, determines its own output path taking as given the fringe's anticipated reaction. Thus, in contrast to a Nash-Cournot world, the cartel is in a position where it can manipulate the fringe's reaction.

Those models have provided extremely important insights into the functioning of imperfectly competitive markets under the dynamic conditions that follow from

10 The well-known special case of a stationary isoelastic demand, under which the monopolistic and competitive price paths coincide (Stiglitz 1976), rests not only on an assumption of zero marginal cost, but also on an assumption that the initial reserves are at the same exogenously given level in both situations (Gaudet and Lasserre 1988).

11 Hotelling analyses in some details the cases of perfect competition and monopoly. He provides no formal analyses of oligopolistic market structures, other than a very sketchy discussion of duopoly. He does recognize, however, that a market composed of 'a few competing sellers' is 'more closely related than either [monopoly or perfect competition] to the real economic world' (Hotelling 1931, 171).

the non-renewability of resource stock and they have been very useful tools of analysis. But they also raise some questions as to the proper way to model non-renewable resource markets. In particular, one drawback from which they suffer is that the strategies are limited to being a function only of time and the initial stocks of the resource – so-called open-loop strategies. This implicitly assumes that the players can credibly commit at the outset to whole-price or output paths. In the absence of the capacity to credibly commit in this way, the equilibria they generate are generally not robust to deviations of the vector of resource stocks from its equilibrium level. To use the game theory jargon, the equilibria obtained are generally not subgame perfect. In the case of the leader-follower approach, the derived equilibria may not even be time-consistent, in the sense that the cartel may, in some cases, wish to deviate from the initially chosen path at some future time if it is allowed to reconsider its plan (Newbery 1981; Ulph 1982; Maskin and Newbery 1990; Karp and Newbery 1992, 1993).¹²

An alternative is to assume that the strategies consist of decision rules that express a producer's rate of extraction as a function of the current stocks of the resource and possibly of time – so-called Markovian strategies, or stationary Markovian strategies if independent of time.¹³ The resulting equilibria are of the feedback (or closed-loop) type and will be Markov perfect: they will constitute a Nash equilibrium not only for the resource stocks along the solution path, but also for all out of equilibrium resource stocks. Solving for Markov perfect Nash equilibria in this context raises very difficult theoretical and computational issues and much remains to be done. Only very recently (Groot, Withagen, and de Zeeuw 2003) has a solution to the feedback Stackelberg equilibrium been derived for the case of linear demand and constant marginal cost.

Another drawback of the dominant firm approach is that not all resource markets can be characterized as composed of a dominant cartel and a fringe. In fact, the market structure of the oil industry is an exception rather than the rule in this respect.¹⁴ Although historically there have been numerous attempts to cartelize other natural resource industries, none have been very successful at controlling the market (Eckbo 1976, Teece, Sunding, and Mosakowski 1993; Suslow 2005; Benchekroun, Gaudet, and Long 2006) and, when able to attain some success in pushing up price, have been short-lived.¹⁵ Certainly, many resource markets can

12 An equilibrium may be time consistent but not subgame perfect: there may not be any incentives to deviate from the initially announced plan if allowed to reconsider in the future, but still the decision may not remain optimal if the vector of resource stocks deviates from its equilibrium value. Of course, if the equilibrium is subgame perfect, then it is also time consistent.

13 This type of strategy is called Markovian because it depends only on the current resource stocks and not on the whole history of the stocks.

14 The diamond industry may be another such exception, the De Beers cartel having exercised strong control over supply for many years. But the structure of that industry is also changing, since it appears that De Beers is having to loosen its grip over supply in the face of emerging competition and government regulations (*Economist*, 24 February 2007, 68).

15 Other than OPEC, the resource cartels studied by Pindyck (1977, 1978) (copper and bauxite) have since completely ceased their activities.

best be viewed as composed of a small number of producers involved in simultaneous decision making, with none of them exerting any dominance, as in the standard Cournot competition.

The properties of static Cournot equilibria are well known. Modelling the non-renewable extraction game as an open-loop dynamic game can be viewed as a direct replication of the Cournot framework in a dynamic setting (Lewis and Schmalensee 1980; Loury 1986; Polasky 1992). Indeed, in terms of information structure, open-loop dynamic games are formally identical to static games, since in the open-loop game the player is allowed to condition his decision only on the initial state and time. From a purely technical point of view, the only difference lies in the fact that in a static game the decision space is the space of real numbers, whereas in the open-loop dynamic game the decision space is the infinite-dimensional space of functions (functions of time in this case).

The fact that in a dynamic context the state evolves over time as a function of the actions taken opens the door to more intricate analysis. Some issues that appear trivial in a truly static context become very relevant in a dynamic context, even if formulated as open-loop. Imagine an industry in Cournot competition which is faced with a temporary exogenous perturbation – for instance, a tax that is imposed for a finite period of time and then lifted. In a truly static world, once this tax is lifted, the production level can be expected to revert to the previous static equilibrium level, since the state is unchanged by the temporary effects of the tax on production. In a dynamic context, however, such as a natural resource industry, the temporary tax will leave the industry in a different state than it would have been without the tax, since current production decisions have an effect on the state, as measured by the resource stocks. If the producers are able to anticipate this temporary perturbation and so are able to condition their open-loop paths on it, then we can expect the open-loop equilibrium production path to be perturbed not only while the tax is in effect, but also before the tax is applied and after it is lifted (Benchekroun and Gaudet 2003).

This example highlights the most objectionable feature of modelling natural resource oligopolistic markets as open-loop equilibria. In the open-loop approach to a non-renewable resource exploitation game, each producer is assumed to choose at the outset the whole time path of extraction of its own resource stock, conditional only on its initial reserves and those of its competitors. It is therefore assumed that the producers do not use information on the evolution of the level of reserves which they observe after initially making their decision on the production path. Thus, if the above tax scheme was unanticipated at the time of deciding on the production path, it is unlikely that the planned open-loop production would still constitute a Nash-equilibrium once the reserves have adjusted to this exogenous shock.

Open-loop strategies are probably not a good description of how oligopolistic natural resource producers make their decision. As in the sequential decision cartel-fringe model just discussed, it seems more reasonable to assume that producers condition their production decisions on the current state of reserves,

rather than simply on the initial reserves and time, and thus use Markovian decision rules. Of course, this assumes that the producers observe the evolution of reserves, which is a realistic assumption given that much information on current stocks is readily available for most resources. We are then looking for a Markov perfect oligopolistic equilibrium, with each producer choosing its decision rule as a function of the state of reserves, taking as given its competitors' decision rules.

The difficult problem of solving for Markov perfect equilibria in oligopolistic non-renewable resource industries remains open, to a large extent.¹⁶ Nevertheless, it is clear that if producers condition their production decisions on the remaining reserves, then Hotelling's rule needs to be modified. Indeed, if a producer knows that his own resource holdings affect a rival's production decision and hence the rival's reserves, which in turn enter into his own decision rule, then there will be a strategic component to the return on holding reserves. This strategic effect will depend, at any given time, on each producer's remaining reserves and his rivals' reserves (see Benchekroun and Gaudet 2003, for an example).

Let us write this strategic component $s(\vec{X}(t), \pi(t))$, where $\vec{X}(t)$ is the vector of stocks remaining at time t . For the sake of the argument, ignore the stock effects on costs and assume a symmetric equilibrium. We might then rewrite Hotelling's rule as

$$\frac{\dot{\pi}(t)}{\pi(t)} + s(\vec{X}(t), \pi(t)) = r. \quad (6)$$

Unfortunately, so little is known about the properties of the decision rules in this type of equilibrium that it is not possible to put a definite sign on this component of the rate of return. It is therefore not possible to predict the effect it should have on the price path. It certainly cannot be ruled out that it is a factor in explaining the flatness of the observed price paths. But much theoretical work needs to be done before its empirical relevance can be ascertained.

6. Uncertainty

I have so far assumed a world of perfect certainty in which wealth can be held either in the form of known natural resource stocks or in the form of a bond

¹⁶ Salo and Tahvonen (2001) have recently proposed a solution to a feedback equilibrium of this sort for the case of a linear demand, under assumptions that unit cost is an affine function of remaining reserves and that it is not profitable to physically exhaust the resource stocks. More explicitly, they solve in that context for an equilibrium in linear strategies, in which each player's production is positively related to his own stock and negatively related to the rivals' stocks. They show that this feedback equilibrium can lead to markedly different conclusions than the open-loop models of Lewis and Schmalensee (1980), Loury (1986) and Polasky (1992), in particular as concerns the evolution of market shares and industry concentration over time. This seems to be attributable in good part to the assumption that production will become unprofitable before the resource stocks are exhausted. It is difficult to compare this feedback solution with the known open-loop solutions, since it rests on *two* different working assumptions: with respect to the type of strategies allowed and with respect to the depletion of the resource stocks.

whose rate of return is exogenously given. This obviously lacks realism if the objective is to explain long-run price behaviour. Resource stocks are not known with certainty, and both extraction and exploration activities provide information about the ultimate size of available stocks.

If, as extraction proceeds, one gains information about the true size of the reserves, then the net price of the resource will reflect the informational value of extraction. As a consequence, it will not be rising at the rate of interest (Kemp 1976; Loury 1978; Gilbert 1979; Kemp and Long 2007). Extraction will proceed faster than otherwise and this will impact on price.

Exploration is also an obvious way of gaining information about the true level of reserves. With uncertain stocks it becomes profitable to carry out exploration activities in order to reduce this uncertainty. Any resulting revision of the estimated reserves will change the basis for the calculation of the asset price (Deshmukh and Pliska 1980; Arrow and Chang 1982; Lasserre 1984). If the estimated reserves are repeatedly revised upwards as a result of the exploration activity, then the predicted rise at the rate of interest of the net price of the resource will be offset by repeated downward adjustments in the net price that serves as a basis for this rise. Hotelling's rule applies between such revisions of reserves, as usual. But if over a long period of time successful exploration activities dominate the unsuccessful ones, then the asset price may appear to be growing on average at less than the rate of interest. Of course, this will reflect itself in a similar way in the time path of the flow price, whose long-run increase will appear dampened by the successive drops in the asset price.

Insecure property rights have also at times constituted another source of uncertainty, which has had an impact on the patterns of extraction and of price. Faced with a threat of expropriation, owners of resource stocks are likely to act as if their discount rate were higher than it would be in the absence of that threat (Long 1975). Leaving the resource *in situ* becomes a risky investment, since the insecure owner has no guarantee that he will be able to fully reap the fruit of his investment. As a consequence, he will tend to shift production from the future towards the present.¹⁷ It has even been argued that the quadrupling of oil prices between 1973 and 1977 should be attributed not so much to the cartelization of the market, but to the resolution of the property rights issue, which definitely shifted from the companies to the host countries by 1973 (Johany 1979; Mead 1979).¹⁸ According to this argument, as the possibility of nationalization became more and more imminent, the oil companies that held concessions in the Middle East reacted by raising their rate of discount. Once the property rights were secured in their favour, leaving the oil in the ground became a more attractive investment

17 The 'overexploitation' that occurs in such a situation is much like the overexploitation that occurs when a resource is exploited in a common property regime, as is the case, for instance, when many oil producers pump from the same oil pool (see Libecap and Wiggins 1984; Libecap 1989; Gaudet, Moreaux, and Salant 2002).

18 See also on this Salant (1982, 9, 14). A closely related type of uncertainty is where agents face the threat of a price collapse due to massive sale of government-held resource stocks (such as gold) at some unknown time in the future (Salant and Henderson 1978).

for some countries than it would have been for the companies facing the threat of expropriation.

More generally, exploitation of the stocks of natural resources and the decisions to invest in such stocks takes place in a risky environment. To illustrate the possible effect of this on the long-run resource price path, consider a stylized economy in which there are two goods and three assets. One of the goods is a non-renewable resource, that serves as an input into the production of the second good, a composite commodity that can be either consumed or accumulated. The accumulated stock of the composite good can be held either in the form of physical capital or in the form of a bond, which yields the exogenously given risk free rate of return, as above. The physical capital can be either devoted to extracting the natural resource or combined with the flow of extracted natural resource to produce the composite good. Both the production process and the extraction process, and hence the cost of producing the composite good and the cost of extracting the resource, are stochastic.¹⁹

Total wealth in this economy is now held in the form of two risky assets (the non-renewable resource and the physical capital) and one risk-free asset (the bond). In addition to choosing a level of consumption, the representative risk-averse consumer decides on the allocation of his portfolio among those three assets. Hotelling's rule will now take the form of the familiar intertemporal equilibrium asset-pricing rule of portfolio theory, adapted to take into account that the holdings of the non-renewable resource stock cannot be increased (Gaudet and Howitt 1989; Gaudet and Khadr 1991) and yield no dividend.

This means that the equilibrium expected rate of change of the *in situ* value of the resource will generally differ from the rate of interest, so as to reflect the relative riskiness of the natural resource as an asset. Suppose unfavourable productivity outcomes in the resource extraction sector are associated with unfavourable productivity outcomes in the production of the composite commodity and similarly for favourable outcomes. Then, on average, the two outcomes tend to reinforce each other, thus increasing the uncertainty in the economy's future capacity to produce the composite good, as perceived from today's standpoint. To the risk-averse representative consumer, this is an undesirable feature of holding the resource stock which must be compensated through a relatively higher rate of return.

On the other hand, suppose that unfavourable productivity changes in the resource sector are typically accompanied by favourable productivity changes in the production of the composite commodity, and vice-versa. It is then expected that when the resource is least easily accessible, it is also the most productive. The two outcomes therefore tend to offset each other and thus to mitigate the uncertainty in the economy's capability to produce the composite commodity.

19 To be more precise, we might assume, as in Gaudet and Khadr (1991), that each of the production processes depends on a parameter that represents the state of productivity in the sector and that the change in those productivity parameters follows an Itô process.

The resource stock is then a relatively less risky asset and thus does not need to exhibit a large expected appreciation in value relative to the rate of return on the stock of physical capital.

For the purpose at hand, a convenient way of expressing the equilibrium asset pricing condition for the resource is in its so-called consumption-beta form (Breedon 1979; Ingersoll 1987). In this form, Hotelling's rule modified to account for uncertain returns becomes (Gaudet and Khadr 1991):

$$\mu_\pi - A(y)\sigma_{\pi y} = r, \tag{7}$$

where μ_π is the expected rate of appreciation of the *in situ* price of the resource, y is consumption, which can be interpreted in this representative consumer framework as per capita consumption, $A(y)$ is the Arrow-Pratt measure of relative risk aversion, and $\sigma_{\pi y}$ is the instantaneous covariance between the rate of change of the *in situ* resource price and the rate of change of consumption.

In a world with many risky assets, we can write this equilibrium condition in a different form. If μ_M is the instantaneous rate of return on any portfolio that has a non-zero covariance with changes in consumption, then we also have $\mu_M = r + A(y)\sigma_{My}$. Hence, by substitution into (7), Hotelling's rule can also be rewritten:

$$\mu_\pi - \beta[\mu_M - r] = r, \tag{8}$$

where $\beta = \sigma_{\pi y}/\sigma_{My}$ is the so-called consumption-beta.

It is now apparent from (7) that the expected rate of appreciation of the *in situ* price will differ from the rate of interest by a risk premium that takes the sign of the covariance between the rate of change in the *in situ* price and the rate of change of consumption. A positive covariance means that the return on the resource stock would tend to accrue in states of the world where marginal utility of consumption is low. This makes the resource a relatively risky asset and calls for a positive risk premium. But this covariance may well be negative, in which case holding the resource stock constitutes a form of insurance against adverse changes in consumption. The risk premium associated with holding the resource stock is then negative, and the expected rate of increase in the *in situ* price will be less than the rate of interest. In fact, if the risk premium is sufficiently negative, we may even expect the *in situ* price to be decreasing.

This has immediate implications for the flow price of the resource, the expected rate of increase of which is now given by

$$\mu_p = \mu_\pi \left(1 - \frac{c(t)}{p(t)} \right) + \mu_c \left(\frac{c(t)}{p(t)} \right), \tag{9}$$

where μ_p and μ_c denote, respectively, the expected rates of change of the flow price and the expected rate of change of extraction cost. If the covariance between the rate of change in the *in situ* price and the rate of change of consumption is

sufficiently negative, both terms on the right-hand side may be negative. Thus, negative risk premia on the holding of resource stocks can reconcile Hotelling's rule with observed price paths that are relatively flat and even decreasing. A direct test of this conjecture would require data on asset prices (π), as opposed to flow prices (p). Unfortunately, only flow prices are observed.

The rare cases where tests of Hotelling's rule have been carried out taking into account the possibility of risk diversification (Young and Ryan 1996; Slade and Thille 1997; Roberts 2001; Livernois, Thille, and Zhang 2006) tend to support the view that this is an important element in the determination of the return of natural resource stocks.²⁰ Its role in explaining the observed behaviour of resource price paths certainly deserves serious empirical investigation.

7. Concluding remarks

In conclusion, I want to emphasize that the foregoing discussion should not be viewed as an exhaustive survey of the field of natural resource economics or of the relevance of Hotelling's rule.²¹ I chose to concentrate on those theoretical developments that seem more likely to reconcile the asset pricing rule that is Hotelling's rule with the observed behaviour of market prices over time. In doing so, I have ignored important issues and topics for which Hotelling's rule is of particular relevance. Let me just mention resources and growth, sustainability, recycling, natural resources in national income accounting, natural resource taxation, investment in extraction capacity, and investment in backstop technologies. I have also ignored the large literature on renewable resources, which, contrary to non-renewable resources, admit a steady state if managed in an appropriate manner. Like non-renewable resources, renewable resources are assets to their owners and are subject to the same asset markets equilibrium conditions, with the important difference that stocks of renewable resources, such as fisheries or forestries, yield a dividend *in situ* in the form of natural growth. Some renewable resources are also more prone to problems raised by difficulties in applying private property rights, which leads to free access or common property exploitation, another issue I have ignored in my discussion of Hotelling's rule.

The last point raises the question of externalities associated with the exploitation and use of natural resources. The global warming debate and the

20 Livernois, Thille, and Zhang (2006) is the only study that actually uses direct observations of the asset price. Their data come from the sales of logging rights for old-growth timber in the U.S. Pacific Northwest. This resource can be considered non-renewable, since the old-growth timber in that region is several hundred years old. Young and Ryan (1996) use aggregate average cost for four Canadian mining industries, while Slade and Thille (1997) use cost estimates for 14 Canadian mining firms. Roberts (2001) uses the flow price to represent the asset price, which amounts to assuming that cost are either zero or a constant proportion of the flow price, so that the rates of change of the two prices are the same.

21 For a more exhaustive survey of the literature on non-renewable resources see (Krautkraemer 1998).

environmental concerns it raises with respect to the use of energy resources is a perfect example of this. From a social point of view, the net benefits will take into account the damage costs that result from the use of those energy resources, in addition to the cost of extracting them. Hotelling's rule then comes through as a social optimality condition, in the form of an intertemporal arbitrage rule with policy implications. It is conceivable that the resulting socially optimal price path will be flatter than it would be in the absence of the externality, or even decreasing, in order to push the lasting damage cost towards the future.²² To the extent that such environmental policies were to be implemented, one should observe the same flattening effects on the actual market price paths.

The impact of environmental policies on market equilibrium resource price paths is obviously an interesting direction for further research. So is the application of Hotelling's rule to the more general analysis of what some perceive as the 'new scarcity' (Simpson, Toman, and Ayres 2005), namely, the limited capacity of the environment as a waste sink.

Appendix: Data sources

Copper, lead, and zinc: Manthy (1978) and U.S. Geological Survey, *Mineral Commodity Summaries*

Tin: Potter and Christy (1962) and U.S. Geological Survey, *Mineral Commodity Summaries*

Aluminum, nickel and silver: U.S. Geological Survey, *Mineral Commodity Summaries*

Coal, natural gas: Manthy (1978) and U.S. Energy Information Administration, *Annual Energy Review 2005* (2006)

Petroleum: Potter and Christy (1962) and U.S. Energy Information Administration, *Annual Energy Review 2005* (2006)

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22 For an interesting first effort at modelling this problem, see Bartelme, Lim, and Yee (2007).

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