1)
(a)

$$\frac{\partial}{\partial x}(\sqrt{x^2+y^2}) = \frac{\partial}{\partial x}((x^2+y^2)^{1/2}) = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot \frac{\partial}{\partial x}(x^2+y^2) =$$
$$\frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

(b)

$$\frac{\partial}{\partial x}(\frac{x+y}{xy-1}) = \frac{\frac{\partial}{\partial x}(x+y) \cdot (xy-1) - (x+y) \cdot \frac{\partial}{\partial x}(xy-1)}{(xy-1)^2} =$$
$$\frac{(xy-1)-(x+y)y}{(xy-1)^2}$$

(c)

$$\frac{\partial}{\partial y}(\sin^2(x-3y)) = 2\sin(x-3y) \cdot \frac{\partial}{\partial y}(\sin(x-3y)) =$$
$$2\sin(x-3y) \cdot \cos(x-3y) \cdot \frac{\partial}{\partial y}(x-3y) = 2\sin(x-3y) \cdot \cos(x-3y) \cdot (-3) =$$
$$= -6\sin(x-3y) \cdot \cos(x-3y)$$

2)
(a)

$$\frac{\partial}{\partial z}(e^{-xyz}) = e^{-xyz} \cdot \frac{\partial}{\partial z}(-xyz) = e^{-xyz} \cdot (-xy)$$

(b)

$$\frac{\partial}{\partial y}(x\sin y\cos z) = x\cos z\frac{\partial}{\partial y}(\sin y) =$$
$$= x\cos z\cos y$$

3) We will prove that

$$\frac{\partial^2}{\partial x\partial y}(\ln(2x+3y)) = \frac{\partial^2}{\partial y\partial x}(\ln(2x+3y))$$

We have

$$\frac{\partial^2}{\partial x\partial y}(\ln(2x+3y)) = \frac{\partial}{\partial x}(\frac{\partial}{\partial y}(\ln(2x+3y)) = \frac{\partial}{\partial x}[(\frac{1}{2x+3y}) \cdot \frac{\partial}{\partial y}(2x+3y)] =$$
$$= \frac{\partial}{\partial x}[(\frac{1}{2x+3y}) \cdot 3] = 3\frac{\partial}{\partial x}(\frac{1}{2x+3y}) = 3\frac{-\frac{\partial}{\partial x}(2x+3y)}{(2x+3y)^2} =$$
$$-3\frac{2}{(2x+3y)^2} = -6\frac{1}{(2x+3y)^2}$$

Similarly,

$$\frac{\partial^2}{\partial y \partial x}(\ln(2x+3y)) = \frac{\partial}{\partial y}(\frac{\partial}{\partial x}(\ln(2x+3y)) = \frac{\partial}{\partial y}[(\frac{1}{2x+3y})\cdot\frac{\partial}{\partial x}(2x+3y)] =$$

$$= \frac{\partial}{\partial y}[(\frac{1}{2x+3y})\cdot 2] = 2\frac{\partial}{\partial y}(\frac{1}{2x+3y}) = 2\frac{-\frac{\partial}{\partial y}(2x+3y)}{(2x+3y)^2} =$$

$$-2\frac{3}{(2x+3y)^2} = -6\frac{1}{(2x+3y)^2}$$

4) I give an example applying the rules of derivation (see mathima 1):

$$\frac{\partial^2}{\partial x \partial y}(\sin(xy)) = \frac{\partial}{\partial x}[\frac{\partial}{\partial y}(\sin(xy))] = \frac{\partial}{\partial x}[\cos(xy)\cdot\frac{\partial}{\partial y}(xy)] =$$

$$\frac{\partial}{\partial x}[\cos(xy)\cdot x] = \frac{\partial}{\partial x}(\cos(xy))\cdot x + \cos(xy)\cdot\frac{\partial}{\partial x}x =$$

$$(-\sin(xy))\cdot\frac{\partial}{\partial x}(xy)\cdot x + \cos(xy) =$$

$$-\sin(xy)\cdot y\cdot x + \cos(xy) = -xy\sin(xy) + \cos(xy)$$

5) Don't solve this exercise.

6) If $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ we will prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(\frac{\partial}{\partial f})$$

$$\frac{\partial}{\partial f} = \frac{\partial}{\partial x}(\frac{1}{\sqrt{x^2+y^2+z^2}}) = \frac{\partial}{\partial x}((x^2+y^2+z^2)^{-1/2}) =$$

$$-\frac{1}{2}((x^2+y^2+z^2)^{-3/2}\cdot\frac{\partial}{\partial x}(x^2+y^2+z^2) =$$

$$-\frac{1}{2}((x^2+y^2+z^2)^{-3/2}\cdot 2x = -x\cdot(x^2+y^2+z^2)^{-3/2}$$

Therefore

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(\frac{\partial}{\partial f}) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(\frac{x}{(\sqrt{x^2+y^2+z^2})^3})$$

$$= \frac{\partial}{\partial x}(x \cdot (x^2+y^2+z^2)^{-3/2}) =$$

$$\frac{\partial}{\partial x}(x) \cdot ((x^2+y^2+z^2)^{-3/2}) + x \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-3/2} =$$

$$(x^2+y^2+z^2)^{-3/2} + x \cdot (-\frac{3}{2})(x^2+y^2+z^2)^{-5/2}\frac{\partial}{\partial x}(x^2) =$$

$$(x^2+y^2+z^2)^{-3/2} - \frac{3}{2}x \cdot (x^2+y^2+z^2)^{-5/2} \cdot 2x =$$

$$(x^2+y^2+z^2)^{-3/2} - \frac{3}{2}x \cdot (x^2+y^2+z^2)^{-5/2} \cdot 2x =$$

$$(x^2+y^2+z^2)^{-3/2} - 3x^2 \cdot (x^2+y^2+z^2)^{-5/2} =$$

$$(x^2+y^2+z^2)^{-3/2}(-1 + 3x^2(x^2+y^2+z^2)^{-1}) =$$

$$(x^2+y^2+z^2)^{-3/2}(\frac{2x^2-y^2-z^2}{x^2+y^2+z^2})$$

Similarly,

$$\frac{\partial^2 f}{\partial y^2} = (x^2+y^2+z^2)^{-3/2}(\frac{2y^2-x^2-z^2}{x^2+y^2+z^2})$$

$$\frac{\partial^2 f}{\partial z^2} = (x^2+y^2+z^2)^{-3/2}(\frac{2z^2-x^2-z^2}{x^2+y^2+z^2})$$

Therefore, if we add all the terms we take 0.